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Traders' behavioral coupling and market phase transition

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HIGHLIGHTS

- Investigate rare market event from the viewpoint of behavioral consensus of market traders.
- Give a mechanism of behavioral consensus: traders' behavioral coupling through "marking" the market index.
- Present three kinds of market phase transition: the degree of behavioral consensus, the correlation of returns of different stock, and the volatility of market index all vary discontinuously with the behavioral coupling strength parameter.

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ABSTRACT

Traditional economic theory is based on the assumption that traders are completely independent and rational; however, trading behavior in the real market is often coupled by various factors. This paper discusses behavioral coupling based on the stock index in the stock market, focusing on the convergence of traders' behavior, its effect on the correlation of stock returns and market volatility. We find that the behavioral consensus in the stock market, the correlation degree of stock returns, and the market volatility all exhibit significant phase transitions with stronger coupling.

1. Introduction

Traders in the Chinese stock market were shocked by the price swings on the first trading day of 2016 when the circuit breaker was triggered twice and trading halted until the stock market closed. In fact, the Chinese stock market experienced violent fluctuations from June 2015 to August 2015. Similar phenomena are not rare in the global stock markets. For example, the US stock market experienced a "flash crash" on May 6, 2010, when sell orders flocked, causing the Dow Jones Industrial Average (DJIA) to drop nearly 5% and at least 30 component stocks of the S&P 500 stock index to decline by 10% or more within 5 min [1]. Even earlier, on October 19, 1987 ("Black Monday"), the DJIA dropped approximately 22.6%.

Although these crashes may be in different forms, they have some common characteristics: first, the original heterogeneous trading behavior trends toward consensus, and sell orders become dominant [2]. This behavioral consensus causes the stock price to fall, or, more seriously, it causes "liquidity black holes" [3]. In fact, some researchers just regard market crash as an event arising from severe mismatch in liquidity [4,5]. Second, when the market is down, the individual stocks would have a higher positive correlation than the normal condition [6,7]. Third, the negative price returns further increase the future volatility, which is so-called leverage effect [8–10]. Reigneron et al. propose that the index leverage effect is stronger than individual stocks because the average correlation between stocks is increasing [11].

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Market crashes can definitely bring huge losses to investors and hit the market sentiment. Therefore, lots of researchers have tried to explore the causes of these stock market disasters and provided empirical or theoretical supports for this issue. According to the efficient market hypothesis, these crashes should be attributed to drastic exogenous fundamental shocks. However, Johansen et al. find that most crashes are associated with an endogenous origin rather than external shocks [12]. Because the exogenous reasons cannot give a persuasive explanation for such sudden systemic collapses of financial systems, some researchers turn to explore endogenous mechanism.

From some scholars' point of view, certain trading strategies are blamed for these crashes. For example, Yim et al. propose a double auction agent-based model and conclude that the strategy of chartists decreases the market stability [13]. Kim and Markowitz use an agent-based model to explore the source of the 1987 crash, and they find that the portfolio insurance strategies destabilize the market [14]. Fagiolo et al. find that high-frequency trading increases market volatility and generates flash crashes by either generating high bid–ask spreads or synchronizing on the sell side of the limit order book [15].

While other scholars give some behavioral or psychological explanations to these crashes. Sornette proposes that large market crashes are just as critical points in the statistical physics, and he emphasizes the positive feedbacks, i.e., self-reinforcement, lead to collective behavior, such as herding in sells during a financial crash [16]. Johansen et al. develop a rational expectation model to study the crash, and make a conclusion that the crash may be caused by the local self-reinforcing imitation among noise traders, which will lead traders to submit the same order(sell) [17]. Westerhoff develops a multi-agent model which contains fundamentalists and chartists, and he find that the chartists will be panic when the stock price drops sharply, then the selling pressure may cause a next panic and finally lead to a severe market crash [18].

In fact, these phenomena can be regarded as "stampedes" (escape panics) in the stock market, somewhat similar to the pedestrian crowds in panic [19,20]. In the situations of escape panics, the pedestrian would get nervous, they all tend to move faster to the exit when compared to the normal situations. And then the interactions among individuals become physical, they start pushing each other, causing jams and large pressure. The escape will slow down when the injured people forming obstacles [21–23]. Similarly, in the condition of market stress, the trading behavior tends to be highly homogeneous, which means the sell side order will be overwhelming. Meanwhile, the traders are more likely to submit the sell order with a lower price in order to "escape" the market quickly. Indeed, we think one of the origins of these phenomena is just traders' behavior are coupled together by the stock index itself. As a matter of fact, Shiller makes some questionnaire surveys to study investors' behavior around the crash of 1987. He finds that investors react to the market drops themselves rather than any other specific news [24]. Besides, we can easily observe that many intraday traders in the real stock market do care about the stock index fluctuation especially when the market drops a lot.

So in this paper we use agent-based modeling to test the impact of the market ups and downs on traders' behavior [25]. We emphasize the traders' behavior will converge spontaneously based on the idea that the traders all take the whole market quotation into account. At last, this behavioral consensus would lead to a bad market liquidity and a strong correlation among individual stocks. High correlation means the stocks tend to go up or down together, so it is inevitable that the market volatility or systematic risk would increase. This paper starts from the coupling behavior based on the stock index and considers how it affects the correlation of stock returns and market volatility. Revealing the mechanism of the behavioral consensus of the traders would be helpful to prevent such financial risks.

2. Market model

We use a multi-agent model based on the double auction mechanism to simulate transactions in a multi-asset market. We have some assumptions in our model:

(1) There are M stocks on the market, and we define a stock index weighted by share capital.

(2) N traders exist, and each trader only trades one stock. Because we do not want to consider the influence of asset portfolio, we assume that each agent chooses one kind of stock to trade randomly at the beginning and fix it in the whole simulation.

(3) Short-selling or margin-buying is not permitted, which means traders cannot sell stocks more than they hold, or submit bids more than they can afford.

(4) Each agent has one opportunity to submit an order in a single trading day, and the type of the order is limit order.

(5) The traders' decisions are equally spaced, and the potential transaction happens instantly.

(6) At the end of a trading day, the unsettled orders are canceled.

Note that some hypotheses in our model seem impractical (for example, one order at most in a single trading day, or equally spaced decision-making), but these assumptions simplify the model design [26] and have little influence on the qualitative conclusion of our concern.

2.1. Trading mechanism

On a typical trading day $t\,(t = 1, 2, \ldots, T)$, the trading sequence is randomly determined. Then agents take turns to submit order according to the determined sequence. Agent i chooses the type of order by the following formula:

$$z_i(\tau; t) = b\upsilon(\tau; t) + (1 - b)\upsilon_i(\tau; t) \tag{1}$$

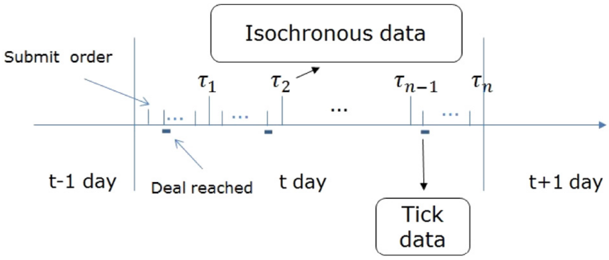

Fig. 1. Tick data and isochronous data. Tick data is obtained when new transaction reached, while isochronous data is updated at fixed time point.

where a positive (negative) $z_i(\tau; t)$ leads to a buy (sell) order. The parameter b shows the index-dependent strength (coupling strength); that is, how much the traders refer to the index-changing trend when making a trading decision. The variable $v(\tau; t) \in \{-1, 0, 1\}$ represents the short-term ups and downs of the market index within a trading day, and the variable $v_i(\tau; t) \in \{-1, 0, 1\}$ is determined by all the other information except the index and assumed as a random variable. The formula (1) means that agents take two factors into account when making trading decisions: the market short-term ups and downs, weighted as b; and the random factor associated with different agents and stocks, weighted as $1 - b$. Note that we record both tick data and isochronous data. Tick data is only updated when a new transaction happens, whereas isochronous data is recorded at regular time intervals, just like the high frequency data of five minutes interval in real stock market. We give a graphic illustration (Fig. 1) to explain it better. Each agent submits order in turns, and if a deal is achieved, it outputs a new tick data, which contains the deal price, transaction volume and updated index. And then we obtain a new isochronous data record after a fixed number of traders Δ have made trading decisions. The data includes the latest price of each stock, the latest stock index and the total transaction volume during the period. In fact, if there is only one stock, we do not need to separate these two kind of data from each other. But in a multi-asset model, if we use the tick data, then we cannot make a horizontal comparison between different stocks in the same time nodes for the reason that most stock prices would not change for most of the time. This is why we record isochronous data. Besides, the newest quotation is updated every few seconds in the real market. In this case, we use isochronous data to judge the market trend. We can simply regard that the tick data is only recorded by the transaction system but unknown to the traders. The public or free information is the isochronous data. $v(\tau; t)$ and $v_i(\tau; t)$ are calculated as follows:

$$v(\tau; t) = sgn(L(\tau; t) - L(\tau - 1; t)) \tag{2}$$

$$v_i(\tau; t) \in \{-1, 0, 1\} \tag{3}$$

where $L(\tau; t)$ denotes the τth stock index of day t, and $L(\tau - 1; t)$ denotes the $(\tau - 1)$th stock index of day t and sgn denotes the sign function. If the market is up in the last time period, then $v(\tau; t) = 1$; if the market is down in the last time interval, then $v(\tau; t) = -1$; if the market is at a fixed state in the last time period, then $v(\tau; t) = 0$. If $z_i(\tau; t) > 0$, agent i would submit a buy order on day t; if $z_i(\tau; t) < 0$, agent i would submit a sell order on day t; if $z_i(\tau; t) = 0$, agent i would not submit any order on day t. The submitted limit price is determined as follows:

$$p_b^i(t) = p(\tau; t)(1 + \delta\eta_1) \tag{4}$$

$$p_a^i(t) = p(\tau; t)(1 + \delta\eta_2) \tag{5}$$

where $p_b^i(t)$ denotes the submitted limit price of agent i on day t if it is a buy order, $p_a^i(t)$ denotes the submitted limit price of agent i on day t if it is a sell order. The parameter δ represents the standard deviation of the price adjusting. $p(\tau; t)$ is the latest price of the stock agent i trades on day t, $\eta_1 \sim N(0, 1)$, $\eta_2 \sim N(0, 1)$. The quantity of the submitted order is determined as follows:

$$Q_b^i(t) \in \{1, 2, \ldots, \min(10, \lceil C^i(t)/p_b^i(t) \rceil)\} \tag{6}$$

$$Q_a^i(t) \in \{1, 2, \ldots, \min(10, Q^i(t))\} \tag{7}$$

where $Q_b^i(t)$ denotes the quantity submitted by agent i on day t if it is a buy order; $Q_a^i(t)$ denotes the submitted limit price of agent i on day t if it is a sell order. They are both randomly selected from the above collections. $Q^i(t)$ denotes the quantity of the stock agent i owns on day t; $C^i(t)$ denotes the quantity of cash agent i owns on day t. Note that we do not consider the underselling in panic in this paper, which means the traders would not sell stocks at low prices. We concentrate on the effect that makes traders submit sell order, so we let the sell price be random.

If an agent submits an order, the order will be sorted by price priority principle. Next, the trading system will match the orders instantly until the orders on the order book cannot be traded anymore, and the latest transaction price, index and agents asset accounts will be updated. Then the next agent will submit an order, and the transaction continues just as the previous process.

2.2. Statistical indicator

The coupling strength b ranges between 0 and 1. We repeat the transaction process at every fixed b to calculate the statistical indicators we need.

The first indicator is the degree of behavioral consensus, D, which describes the consistency of the type of the agents' order. The formula is as follows:

$$D = \left| \frac{N_+ - N_-}{N_+ + N_-} \right| \tag{8}$$

where $N_+(N_-)$ denotes the number of traders submitting buy (sell) order in a trading day. Then we calculate the average D in all trading days and obtain the degree of behavioral consensus at the fixed b.

The second kind of indicators are designed to describe the correlation of stock returns. Based on the hypothesis that agents trade equally spaced, we obtain isochronous data. Then we serialize the different intraday data and calculate the stock correlation matrix at different coupling strength b. Then we can get the average correlation ρ. The formula is as follows:

$$\rho = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j \neq i, j=1}^{M} \rho_{ij} \tag{9}$$

where ρ_{ij} denotes the Pearson correlation coefficient of the returns of stock i and stock j. Note that the stock return is log-return and computed from time series of stock price or stock index (isochronous data). Besides, we calculated the maximum eigenvalue of the correlation matrix and the variance contribution rate of the common factor associated with the maximum eigenvalue. According to the random matrix theory, we know that the biggest eigenvalue of empirical stock correlation matrix contains most information quantity [27–29]. And according to the factor analysis, the factor associated with the biggest eigenvalue is the most important common factor which can explain most of the variance of the original normalized data [30]. The variance contribution rate of the common factor associated with the maximum eigenvalue is calculated by the following formula:

$$a = \frac{\lambda_1}{\sum_{i=1}^{M} \lambda_i} \tag{10}$$

where a denotes the variance contribution rate, λ_i represents the eigenvalue of the matrix and λ_1 is the maximum eigenvalue.

In addition to the indicators mentioned above, we also consider the market volatility. The volatility is calculated as follows:

$$s = \sqrt{\frac{1}{K-1} \sum_{k=1}^{K} (r_k - \bar{r})^2} \tag{11}$$

where s denotes the volatility based on isochronous data. K represents the total number of the sample data, which means the population of isochronous data.

3. Analytic results

Before the simulation, we first try to get some analytic results. The basic idea of the analysis is as follows: first, find the functional relationship between the probability of submitting orders and the coupling strength b by the whole probability formula; second, the differences of the probabilities of submitting different types of orders in different ranges of b mean a different b has a different effect on the degree of behavioral consensus D, the correlation of stock returns ρ and market volatility s.

Assume that the probabilities of up-trending, down-trending or staying at the same state when comparing the latest index with the previous isochronous data are p_1, p_2 and p_3, respectively, $p_1+p_2+p_3 = 1$. The probabilities of the positive, negative and neutral expectations that are affected by random factors are $P\{v_i(\tau; t) = 1\}$, $P\{v_i(\tau; t) = -1\}$ and $P\{v_i(\tau; t) = 0\}$, respectively. Then we obtain $P\{z_i(\tau; t) > 0\}$, $P\{z_i(\tau; t) < 0\}$ and $P\{z_i(\tau; t) = 0\}$, which signify the probability of submitting a buy order, sell order or no order, respectively. $b \in [0, 1]$, and $v(\tau; t)$, $v_i(\tau; t)$ can be -1, 0 or 1. The derivation process and results in detail are listed in the Appendix.

When b changes to a different range, the probability of submitting orders is different. Thus, we summarize the different probability distributions of submitting orders when b in different ranges; the result is illustrated in the first four columns of Table 1. Assume that $P\{v_i(\tau; t) = 1\} = P\{v_i(\tau; t) = -1\} = P\{v_i(\tau; t) = 0\} = 1/3$; then we calculate D according to the Law of Large Numbers in the fifth column of Table 1.

According to simple algebraic manipulation, we find that $D\{b = 1\} > D\{b \in (0.5, 1)\} > D\{b = 0.5\} > D\{b \in (0, 0.5)\} > D\{b = 0\}$. When $b = 0$, the consistency of orders is close to zero. The trading decisions are decided by random factors rather than coupling to the index, and the numbers of agents submitting sell orders or buy orders are nearly the same. When $0 < b < 0.5$, although we do not know the probabilities of the entire market rising or falling, the requirement that $p_1 + p_2 + p_3 = 1$ and p_1, p_2 and p_3 are non-negative makes D relatively small. And $b = 0.5$ is the tipping point of the degree

Table 1
Probability of submitting orders and the degree of behavioral consensus in different b.

b	$P\{z_i(\tau;t)>0\}$	$P\{z_i(\tau;t)<0\}$	$P\{z_i(\tau;t)=0\}$	D
{0}	$P\{v_i(\tau;t)=1\}$	$P\{v_i(\tau;t)=-1\}$	$P\{v_i(\tau;t)=0\}$	0
(0,0.5)	$p_1P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=1\}$	$p_2P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=-1\}$	$p_3P\{v_i(\tau;t)=0\}$	$\frac{\lvert p_1-p_2\rvert}{p_1+p_2+2}$
{0.5}	$p_1P\{v_i(\tau;t)=0\}$ $+P\{v_i(\tau;t)=1\}(p_1+p_3)$	$p_2P\{v_i(\tau;t)=0\}$ $+P\{v_i(\tau;t)=-1\}(p_2+p_3)$	$p_1P\{v_i(\tau;t)=-1\}$ $+P\{v_i(\tau;t)=1\}p_2+P\{v_i(\tau;t)=0\}p_3$	$\lvert p_1-p_2\rvert$
(0.5,1)	$p_1+P\{v_i(\tau;t)=1\}p_3$	$p_2+P\{v_i(\tau;t)=-1\}p_3$	$P\{v_i(\tau;t)=0\}p_3$	$\frac{3\lvert p_1-p_2\rvert}{p_1+p_2+2}$
{1}	p_1	p_2	p_3	$\frac{\lvert p_1-p_2\rvert}{p_1+p_2}$

Table 2
Parameter setting in the simulation.

Parameter	Value	Description
N	2000	Number of agents
M	5	Number of stocks
T	500	Total trading day
δ	0.005	Standard deviation of the price adjusting
Δ	20	Number of agents making trading decisions in an interval
P_{j0}	100	Initial price of stock j
$C^i(0)$	100*{1, 2, ..., 10}	Initial cash endowment of agent i
$Q^i(0)$	{1, 2, ..., 10}	Initial stock endowment of agent i

of behavioral consensus. When $0.5 < b < 1$, the agents behavior is more strongly coupled by the index. Heterogeneous trading behavior gradually converges to homogeneous trading behavior, and the consistency of the type of orders becomes more obvious. Moreover, we know that when $b = 1$, the degree of behavioral consensus will have a jump discontinuity.

Regarding the correlation of stock returns and the volatility, it is difficult to make a quantitative derivation. But we know that these statistical indicators depend on the probability distribution function of $z_i(\tau; t)$. Moreover, the probability distribution function of $z_i(\tau; t)$ is different when b in different range. Therefore, the different probability distribution can lead to a phase transition of the indicators when $b = 0.5$.

4. Simulation results

The parameter setting of the simulation is shown in Table 2.

We first obtain a representative time series when b is fixed. Fig. 2 shows the stock price time series and stock index time series when $b = 0.7$. In this case, the argument b is relatively large, so the market trend would greatly influence the traders' behavior and lead to behavioral consensus. As a consequence, the trend of the market index and the individual stocks would have a high consistency. We also conduct a simulation with $b = 0.3$, and the result is markedly different. This time, we find that the trend of the index and the individual stock would have a lower consistency because if b is relatively small, the trading behavior would be mainly affected by random factors. With this condition, traders' behavior has a high degree of heterogeneity.

We calculate the three kinds of statistical indicators mentioned above after some simulations. Finally, we find that the results of the simulations confirm the analytic analysis. The degree of behavioral consensus, the correlation of log-returns and volatility have an obvious phase transition when $b = 0.5$.

Fig. 3 illustrates the relationship between the degree of behavioral consensus and the coupling strength. When $0 < b < 0.5$, the degree of behavioral consensus is very slight (close to zero). This result occurs because traders' behavior mainly depends on individual random factors; thus, the types of orders are highly inconsistent. When $0.5 < b < 1$, the degree of behavioral consensus rises significantly. This result occurs because traders' behavior mainly depends on a common factor (index fluctuation), and there is a high correlation between the expectation of individual stocks and latest trend of the index. As a result, the types of orders are highly consistent. We highlight the situation when $b = 1$. In this case, traders' behavior totally depends on the index trend, leading to the same type of orders during a period while the order book in another direction is empty. This result means the market totally loses liquidity. According to the assumption of the model, when $b = 1$, if the price does not change, then traders will not submit any order. It is meaningless to discuss traders' behavior in this situation, so the figure does not show the point $b = 1$. It is the same as the subsequent figures.

The relationships between the correlation of returns and the coupling strength are illustrated in Fig. 4. When $b = 0$, the types of orders depend on random factors, thus resulting in a more random stock price series. The returns are also more random. Therefore, individual stock returns have a lower correlation. When $0 < b < 0.5$, although traders' strategy partly depends on index factor, we know from the theoretical analysis that the degree of behavioral consensus will slightly increase. That is, one type of orders are a little bit more than the other, leading to a higher consistency of stock sequences and index trend, and a slight rise in the correlation of stock returns. When $0.5 < b < 1$, the consistency of the type of orders is higher, and agents' behavior mainly depends on the market trend. When the index is up (down), the possibility of submitting a buy (sell) order will increase, leading to a general rise (fall) in price. The correlation of individual stocks would significantly

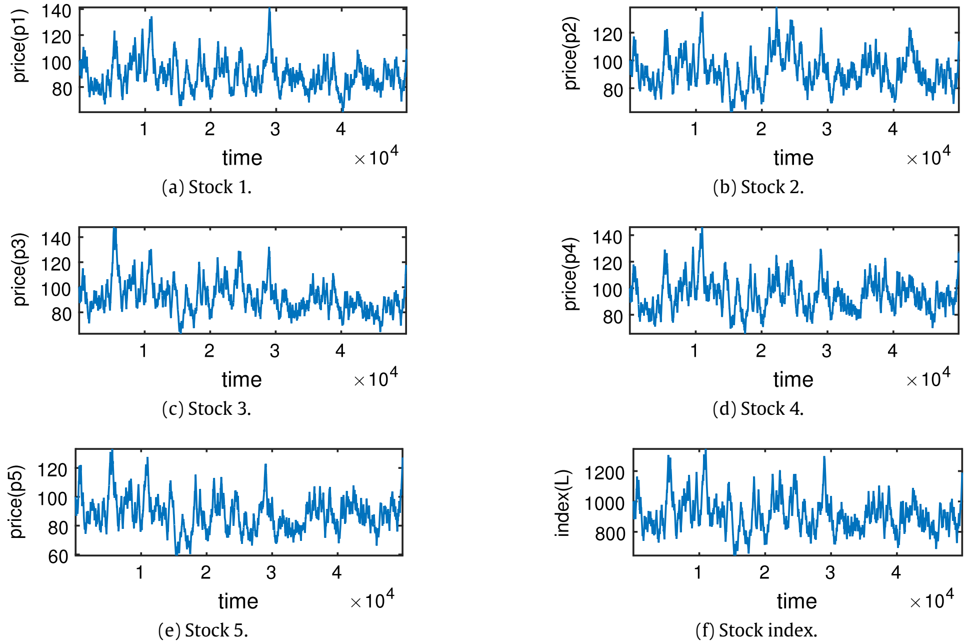

Fig. 2. Time series of 5 stocks and the market index when coupling strength $b = 0.7$. (a)–(e) are the price sequences of the five stocks, and (f) is the market index sequence.

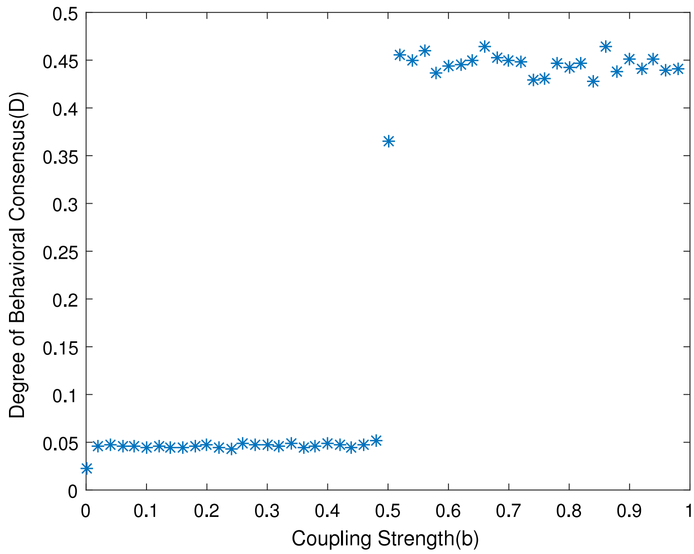

Fig. 3. Relationship between the degree of behavioral consensus and the coupling strength. When $0 < b < 0.5$, the degree of behavioral consensus is slight; when $0.5 < b < 1$, the degree of behavioral consensus is high.

increase. Besides, we go step further to mine the information of the correlation matrix. Although the most important common factor is somewhat abstract in factor analysis, we can infer it is just the index fluctuation that influences all the individual stocks in our model. We show the relationship between the maximum eigenvalue of the stock correlation matrix (after normalizing the original return data) and the coupling strength (Fig. 5). The result is consistent with the average correlation of stock returns. The maximum eigenvalue has a jump at the point $b = 0.5$. And we can see the factor (the index) explains almost 40% of the variance when b is greater than 0.5.

The relationships between the market volatility and the coupling strength are illustrated in Fig. 6. In fact, the market volatility can be calculated by the individual stock weight vector and the correlation coefficient matrix just as the mean–variance analysis in the portfolio theory [31]. So we think the growing index volatility results from the increasing correlation

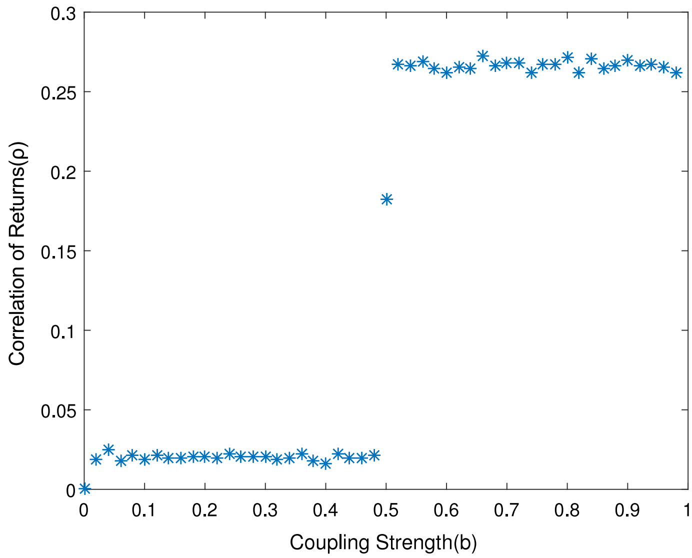

Fig. 4. Relationship between the correlation of returns and coupling strength. When $0 < b < 0.5$, the correlation of stock returns is low; when $0.5 < b < 1$, the correlation of stock returns is greater.

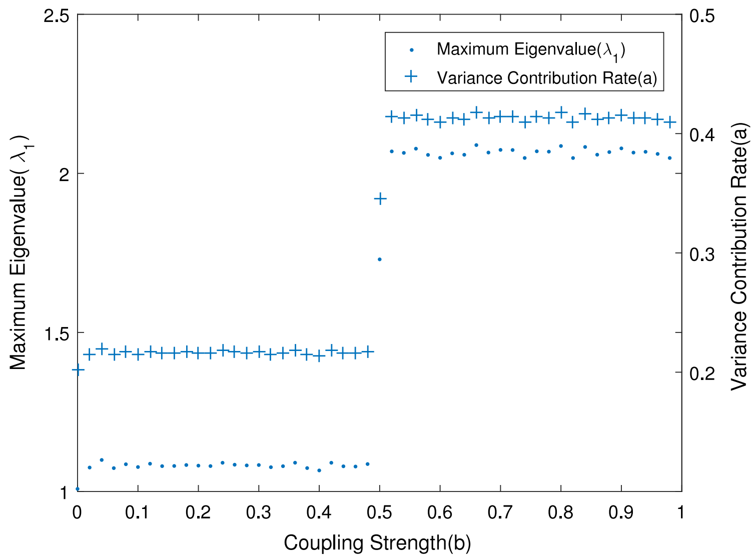

Fig. 5. Relationship between the maximum eigenvalue of the stock correlation matrix and coupling strength. Besides, we show the variance contribution of the common factor(index fluctuation) associated with the maximum eigenvalue. When $0 < b < 0.5$, the maximum eigenvalue and the variance contribution are small; when $0.5 < b < 1$, the maximum eigenvalue and the variance contribution are greater.

of stocks. The correlation of quotations can reflect to the market volatility. Because a higher correlation means the different stocks tend to go up or down together, so the same historical index trend will largely reflect to the later index ups and downs, leading to a greater volatility. Therefore, when $b = 0.5$, the volatility would have a phase transition.

5. Conclusion

In this paper, we simulated the trading in the stock market using an agent-based model and analyzed the inherent mechanism of behavioral consensus in a multi-asset market. Moreover, we discussed its impact on the correlation of returns and market volatility. We designed a parameter b to measure the degree to which the traders are affected by index (coupling strength) and analyzed its relationship with the degree of behavioral consensus, the correlation of stock returns, and market volatility.

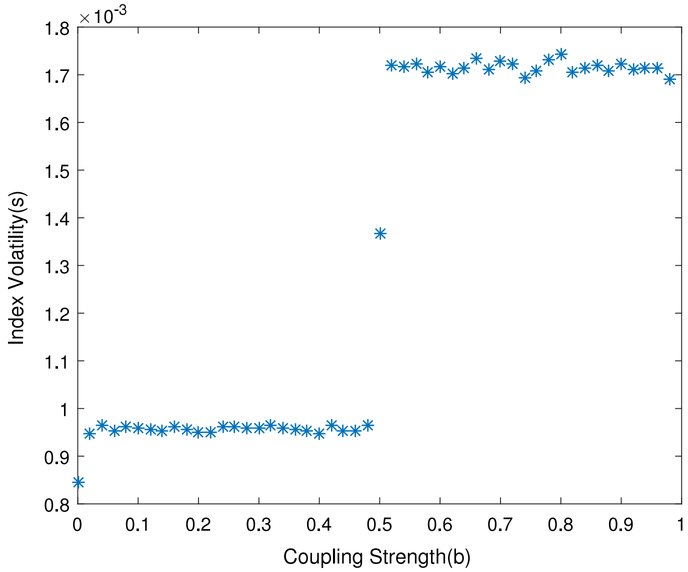

Fig. 6. Relationship between volatility and coupling strength. When $0 < b < 0.5$, and the market volatility is small; when $0.5 < b < 1$, the market volatility is larger.

The results show that these three indicators all have a phase transition at the point $b = 0.5$. When the coupling strength is larger, the degree of behavioral consensus, the correlation of stock returns and market volatility all increase. According to the analytic analysis, convergence is inevitable and would lead to a large change in market quotations. In particular, when $b = 1$, the traders neglect their own random factors and follow the market trend. This effect will lead to a loss of market liquidity.

But we should emphasize that we care about the market phase transition (regime shift) more than the tipping point itself, though we conclude the threshold value is $b = 0.5$. Because in actual trading when traders make a transaction decision they do care the specific degree of the fluctuation, so $v(\tau; t)$ and $v_i(\tau; t)$ are not limited to $\{-1, 0, 1\}$ and buy–sell direction are not simply determined by positive–negative sign of $z_i(\tau; t)$. This means that tipping point in real market may be other specific value except for 0.5.

Overall, our results provide a deep insight into the mechanism of behavioral consensus. It is important for us to develop an early warning signal to suppress the convergence process and prevent extreme market events. Currently, we only discussed the relationship between the degree to which traders refer to the index and the behavioral consensus. It is meaningful for us to understand the convergence phenomenon, but it is still a preliminary work. In a further study, we will extend the model by introducing an adaptive coupling strength b, which has a dynamic adjustment according to different market scenarios.

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Appendix

The probability agent i submits a buy order is as follows:

$$\begin{aligned}&P\{z_i(\tau;t)>0\}=P\{z_i(\tau;t)>0|v(\tau;t)=1\}P\{v(\tau;t)=1\}+P\{z_i(\tau;t)>0|v(\tau;t)=-1\}\\&P\{v(\tau;t)=-1\}+P\{z_i(\tau;t)>0|v(\tau;t)=0\}P\{v(\tau;t)=0\}\\&\quad=P\{b+(1-b)v_i(\tau;t)>0\}p_1+P\{-b+(1-b)v_i(\tau;t)>0\}p_2+P\{(1-b)v_i(\tau;t)>0\}p_3\end{aligned}$$

Then, we discuss the expression $P\{b + (1 - b)v_i(\tau; t) > 0\}$, $P\{-b + (1 - b)v_i(\tau; t) > 0\}$ and $P\{(1 - b)v_i(\tau; t) > 0\}$.

$$P\{b+(1-b)v_i(\tau;t)>0\}=P\left\{v_i(\tau;t)>-\frac{b}{1-b}\right\}=\begin{cases}P\{v_i(\tau;t)=1\} & b=0,\\ P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=1\} & 0<b\le 0.5,\\ 1 & 0.5<b\le 1.\end{cases}$$

$$P\{-b+(1-b)v_i(\tau;t)>0\}=P\left\{v_i(\tau;t)>\frac{b}{1-b}\right\}=\begin{cases}P\{v_i(\tau;t)=1\} & 0\le b<0.5,\\ 0 & 0.5\le b\le 1.\end{cases}$$

$$P\{(1-b)v_i(\tau;t)>0\}=\begin{cases}P\{v_i(\tau;t)=1\} & 0\le b<1,\\ 0 & b=1.\end{cases}$$

After simplifying, $P\{z_i(\tau;t)>0\}$ can be written as a piecewise function as follows:

$$P\{z_i(\tau;t)>0\}=\begin{cases}P\{v_i(\tau;t)=1\} & b=0,\\ p_1P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=1\} & 0<b<0.5,\\ p_1P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=1\}(p_1+p_3) & b=0.5,\\ p_1+P\{v_i(\tau;t)=1\}p_3 & 0.5<b<1,\\ p_1 & b=1.\end{cases}$$

Similarly, the probability that agent i submits a sell order is as follows:

$$P\{z_i(\tau;t)<0\}=\begin{cases}P\{v_i(\tau;t)=-1\} & b=0,\\ p_2P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=-1\} & 0<b<0.5,\\ p_2P\{v_i(\tau;t)=0\}+P\{v_i(\tau;t)=-1\}(p_2+p_3) & b=0.5,\\ p_2+P\{v_i(\tau;t)=-1\}p_3 & 0.5<b<1,\\ p_2 & b=1.\end{cases}$$

The probability that agent i will not submit any order is as follows:

$$P\{z_i(\tau;t)<0\}=\begin{cases}P\{v_i(\tau;t)=0\} & b=0,\\ p_3P\{v_i(\tau;t)=0\} & 0<b<0.5,\\ p_1P\{v_i(\tau;t)=-1\}+P\{v_i(\tau;t)=1\}p_2+P\{v_i(\tau;t)=0\}p_3 & b=0.5,\\ P\{v_i(\tau;t)=0\}p_3 & 0.5<b<1,\\ p_3 & b=1.\end{cases}$$

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