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REGULAR ARTICLE

Asset diversification and systemic risk in the financial system

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Abstract In this study, we have developed a complex network system from the obligation links among banks and links created by portfolio overlaps to simulate the behavior of the financial system. In the network system, we adopt a dynamic allocation mechanism of liquidity to cope with external shocks of liquidity to the bank system. This dynamic mechanism introduces a reinforcing feedback that represents the cycle of assets and liabilities, emphasizing the effect of asset diversification (interbank and external asset diversification). Our results show that the financial system is "robust-yet-fragile" with asset diversification: for small external liquidity shocks, both interbank and external asset diversification can contribute to reducing individual risk and stabilizing the system, whereas for large liquidity shocks, high diversification amplifies the initial impact and destabilizes the entire system. In other words, high diversification can promote liquidity allocation and risk sharing in normal times but amplify the initial shock and engender endogenous systemic crisis in times of distress. This result indicates that diversification is a trade-off between individual risk and systemic risk and is a double-sided sword to risk management of the financial system.

Keywords Complexity in financial system · Systemic risk · Complex network · Liquidity allocation · Diversification

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1 Introduction

The financial crisis of 2007 vividly demonstrates a typical case of systemic risk, which is the risk encountered by the system as a whole. In addition, it correlates with bankruptcy in financial institutions (Kaufman and Scott 2003). Because these institutions are linked by financial obligations, the distress of one operator is transmitted to other operators in the network and on a sufficiently large scale leads to "systemic failure" (Colander et al. 2009).

As noted by Iori et al. (2006), the financial system is based on obligation links among financial institutions. Three types of financial obligations can serve as sources and channels for transmitting financial distress. First, certain households withdraw funds, which can spur self-increased panic and herd behavior among depositors (Diamond and Dybvig 1983; Calomiris and Kahn 1996). Second, banks invest in the same or similar external assets. The failure of one bank can lead to a depreciation of its external assets and can affect the solvency of other banks that hold the same assets (Edison et al. 2000; Kiyotaki and Moore 2002). The third channel is the interbank market (Allen and Gale 2000). On the one hand, the interbank network can be a security guard against liquidity requests for individual banks. This interbank network helps transfer liquidity needs from liquidity surplus banks to liquidity shortage banks. On the other hand, interbank exposure is a channel for contagion when certain institutions become insolvent. Through positive feedback in assets (Brunnermeier 2009), liabilities (Bernanke et al. 1999) and confidence of the entire system (Arinaminpathy et al. 2012), risk in this system will continually grow. Therefore, a financial crisis does not necessarily result from a large impact from outside (exogenous crisis). It is entirely possible for a crisis to "emerge" via local impact and positive feedback effects (endogenous crisis).

This paper relates to various strands of the literature, given that many publications on financial networks have been presented over the past several years. An overview of the existing literature can be found in Allen et al. (2010). First, much of the empirical literature on network structures of interbank markets show that they exhibit characteristics of complex networks. Soramäki et al. (2007) show that the network of interbank payments transferred between commercial banks through the Fedwire Funds Service exhibits a scale-free topology. Power-law degree distribution in economics is also shown by Souma et al. (2003) and Inaoka et al. (2004). In accordance with the corresponding empirical literature, our study of interbank diversification is focused on interbank diversification in a scale-free network and, more specifically, in the scale-free network generated by the Barabási–Albert method (Barabási and Albert 1999) (refered as BA method) and gravity models (Montagna and Lux 2013). Because the random network is a common model assumption in the theoretical literature, we include the random network as a benchmark for comparison.

Second, dual effects of the interbank network have been shown in the related literature, including Gai and Kapadia (2010), Battiston et al. (2012) and Cifuentes et al. (2005). For small shocks, high interconnectivity helps stabilize the system, whereas for large shocks, high interconnectivity amplifies the initial impact (Ladley 2013).

Third, this paper is related to the literature on diversification of external assets in the financial system. In fact, diversification is a trade-off between individual and systemic risk (Corsi et al. 2016). The option of portfolios for different institutions to maintain the smallest individual risk also constitutes an infection channel for an exogenous systemic crisis, as shown in Caccioli et al. (2009) and Greenwood et al. (2015).

However, we find that most of the literature on this issue is based on the *domino effect* and on principles of balance sheet insolvency: for the interbank market, one bank whose liabilities exceed assets initially becomes insolvent and causes losses to its creditors. In turn, its creditors could default on their creditor banks and so on (Gai and Kapadia 2010). Similarly, common asset holding (Huang et al. 2013) can also result in a chain effect of bankruptcy.

Insolvency pertains to the inability of a debtor to pay its debt, and two basic forms of insolvency exist in law: balance sheet insolvency and cash flow insolvency. Balance sheet insolvency occurs when a company's liabilities are greater than its assets. Cash flow insolvency (also referred to as a liquidity crisis) involves a lack of liquidity to pay debts as they come due. A financial institution may show a negative net value on its balance sheet but can remain cash flow solvent if it is able to meet its debt obligations. In fact, in the formation and spread of a financial crisis, cash flow insolvency is a more direct cause of bankruptcy.

The "domino effect" based on balance sheet insolvency is more similar to the chain of consequences of one bank's default on its direct creditors. However, a cyclical entanglement of assets and liabilities can occur during the liquidation process. The ability of a bank to fulfill its liquidity needs for its creditors depends on not only its current cash and asset levels but also its debtors, given that its debtors determine the realizable value at disposal. Liquidity continually allocates throughout this process until the system achieves an equilibrium state. Therefore, from the view of economic practice, using the cash flow standard should be a more meaningful choice. This paper discusses the interbank network from the perspective of cash flow insolvency and extends the literature by considering the impacts of the size of an initial shock and the structure of interbank networks on financial stability.

The framework that we develop here builds on Eboli (2010). The financial network can be viewed as a flow network, a weighted and directed graph endowed with source and sink nodes. A dynamic configuration of liquidity can be viewed as an abstract process of flow diffusion. Banks encountering a shortage of liquidity withdraw from an interbank network and sell their external assets when necessary according to their preferences of liquidation. After the flow diffusion, the call of liquidity transmits and is absorbed into the financial system until a steady state is reached. Banks whose overall liquidity is insufficient to meet their needs will default.

We primarily focus on the role of asset diversification in systemic risk. In the field of finance, diversification involves reducing unsystematic risk by investing in a variety of assets; it is one general technique for reducing investment risk. "Do not put all your eggs in one basket" is a simple example of diversification. However, considering systemic risk, diversification may not always be a suitable choice for risk management.

We base our study of interbank diversification on a random network, scale-free networks generated by the BA model and the gravity model. In addition to the percentage of bankruptcy, we use the shortage of liquidity as an index for systemic risk. We use a static financial network and fixed balance sheet to discuss diversification of interbank and external assets. We emphasize the following important results.

First, we find that the network of interbank markets can have a dual effect. An interbank network can serve as mutual insurance and as a channel for transmission during liquidity crises. When the initial liquidity shock is weak, the proportion of bankruptcy decreases as the average degree increases. Conversely, when the shock is sufficiently large, the proportion of bankruptcy increases as the average degree increases. Our results confirm the conjecture of the Executive Director for Financial Stability of the Bank of England. A robust-yet-fragile property for the financial system is shown; in normal times, connections between financial institutions lead to enhanced allocation of liquidity and increased risk sharing (Haldane 2009). However, in times of crisis, the same interconnections can amplify initial shocks such as the insolvency of a large and highly interconnected bank. Battiston et al. (2015) show how small errors in the knowledge of the contract network of financial institutions can lead to large errors in the probability of systemic defaults. This erroneous result is the “price” of complexity in financial networks.

However, considering unrealized needs of liquidity, the shortage of liquidity in the random network, surplus-centered scale-free network and interbank network generated by the gravity model will decrease, whereas the shortage of liquidity in the deficit-centered scale-free network will increase. This phenomenon is mainly caused by different interbank topologies. Liquidity shocks are evenly absorbed in a random network, whereas shocks are transmitted to the hub nodes in a scale-free network. The hub nodes in a scale-free network are of systemic importance.

Second, diversification in a bank–asset network has a similar effect. Diversification actually involves a trade-off between individual and systemic risk. A bank lowers its own probability of failure by diversifying its portfolios. Holding assets evenly prevents the dramatic devaluation of certain investment failures. However, when many banks diversify their risks in a similar manner, portfolio overlaps will create links among banks. Banks are more likely to be affected by different assets and other banks. When the shock of liquidity is large, the probability of large systemic failures can increase in a more connected bank–asset network.

This paper proceeds as follows. Section 2 characterizes the financial system as a flow network and introduces the initialization of bank balance sheets. Section 3 outlines the liquidity dynamic allocation model. Section 4 describes the simulation results on interbank assets diversification. Section 5 describes the simulation results on external assets diversification. Section 6 concludes the paper.

2 Financial system

The financial system examined here is composed of three parts: banks, households, and external assets. Banks are directly or indirectly connected to one another by financial obligations. Each bank is characterized by its own balance sheet. For assets, let a_i be the value of the external assets of bank i, and let c_i be the cash held by that bank. In addition to external assets a_i and cash c_i, a bank has interbank assets $b_i = \sum_{j \neq i} w_{ji}$. Regarding liabilities, let $d_i = \sum_{j \neq i} w_{ij}$ be the debts owned by other banks, and let h_i be household deposits. Finally the capital (net worth) of bank i is v_i. For each bank,

Table 1 Balance sheet of a bank

Assets	Liabilities
Cash	Household deposits
c	h
Interbank assets	Interbank liabilities
b	d
External assets	Capital
a	v

total assets are equal to total liabilities, which means $v_i + h_i + d_i = a_i + b_i + c_i$. The balance sheet of a bank is shown in Table 1.

In our model, interbank assets, represented by b_i are equally distributed among one's debtor banks. Although this assumption is stylized, it serves as a benchmark for discussion. We use the Erdös–Rényi method (Erdös and Rényi 1960) for random networks. Given the average degree m and total number of banks, every two banks are connected with the same probability m/N. Additionally, we use the Barabási–Albert method (Barabási and Albert 1999) to generate a scale-free network. Each time, one new node is added and m edges are connected from this new node to existing nodes through preferential attachment. The process of financial initialization is as follows:

(1) Provide the total assets for each bank V_i and interbank connection network $\tilde{w}_{ij}$.
(2) The γ_b of assets are interbank assets b_i.
(3) Interbank assets are evenly distributed across in-degree banks; in turn, we obtain the weighted network of interbank connection w_{ij}. Additionally, interbank liabilities, represented by d_i are endogenously determined according to the interbank network.
(4) Based on the capital assets ratio γ_v, we obtain the capital $v_i = \gamma_v V_i$.
(5) Household deposits compensate for the gap in the liabilities side, $h_i = V_i - d_i - v_i$.
(6) According to household deposits h_i and the deposit reserve ratio γ_s, the quantity of cash c_i is determined from $c_i = \gamma_s h_i$.
(7) External assets a_i compensate for the gap in assets, $a_i = V_i - b_i - c_i$.

When initializing the financial system according to the above process, those banks with a large degree in a scale-free network have large interbank liabilities; consequently, they are deficits in the interbank market ($d_i > b_i$). However, we can switch steps (2) and (3) in this process; that is, the γ_b of liabilities are interbank liabilities d_i, interbank liabilities are evenly distributed among out-degree banks, and interbank assets b_i are endogenously determined. Banks that have a large degree within a scale-free network have large interbank assets; in such case, they are surpluses in the interbank market ($b_i > d_i$). To study the effects of the net worth position (deficit or surplus) on the interbank market, we retain the above two initialization methods and call them a deficit-centered scale-free network and surplus-centered scale-free network, respectively. Furthermore, we consider the heterogeneity of total assets among banks and that banks are more inclined to borrow money from or lend money to larger banks. In accordance with Montagna and Lux (2013), we apply the gravity model to generate the banking system.

We assume that the distribution of the size of banks V_i follows a power law. Additionally, we use the probability function

$$p_{ij} = p(V_i, V_j) \propto V_i V_j$$

for the generation of links. Therefore, we obtain the network of interbank connection $\tilde{w}_{ij}$,

$$\tilde{w}_{ij} = \begin{cases} 1, & \text{with probability } \ p_{ij} \\ 0, & \text{with probability } \ 1 - p_{ij} \end{cases} \tag{1}$$

From the interbank assets b_i and network of interbank connection $\tilde{w}_{ij}$, we obtain the weighted network of interbank connection:

$$w_{ij} = \frac{b_i p_{ij}}{\sum_{j \in \Omega_i} p_{ij}},$$

where Ω_i denotes the set of nodes for which $\tilde{w}_{ij} = 1$. This model reproduces the power law in degree distribution, and banks with a large degree in the interbank network can be either deficits or surpluses in interbank markets. We call this an interbank network formed by the gravity model. We therefore obtain four types of networks: a random network, a deficit-centered scale-free network, a surplus-centered scale-free network, and an interbank network created based on the gravity model.

3 Dynamic allocation mechanism of liquidity in the financial system

The initial withdrawal of deposits is the liquidity shock for bank i. The bank will first use its cash and then withdraw from its debtor banks when needed. Furthermore, after the above two processes occur, if bank i cannot cope with liquidity withdrawals, it must liquidate part or all of its external assets. The illiquidity of the market is also considered. When a bank engages in a fire sale or in the premature liquidation of external assets, it may need to accept a depreciated price α. Furthermore, one bank cannot borrow from another financial institution to meet its liquidity needs.

Note that because interbank assets for one bank are the liabilities of another bank, a call from a creditor bank will increase the needs of liquidity from the debtor bank. This knock-on process will continue until the needs of liquidity generated from withdrawals of deposits are compensated for the cash and external assets of banks. Thus, liquidity is transferred throughout the financial system until an equilibrium state is reached. From the view of the diffusion of flow, the source of the flow is the liability, and the sink of the flow is the asset for this process. The diffusion of flow continues to occur until all flows from the source are absorbed by the sink (Eboli 2010). The process of a dynamic transfer mechanism of liquidity is as follows:

(1) For the first round, set the liquidity outflow from a bank i to be initial withdrawal of deposits $\sigma_i = \Delta h_i$;
(2) Compute the cash paid:

$$\Delta c_i = min(c_i, \sigma_i);$$

(3) Compute the ratio of interbank withdrawals:

$$\eta_i(\sigma_i) = min\left[max\left(\frac{\sigma_i - \Delta c_i}{b_i}, 0\right), 1\right];$$

(4) Update the outflow of liquidity from a bank: initial household withdrawals plus withdrawals from its creditor banks.

$$\sigma_i = \Delta h_i + \sum_j \eta_j w_{ij};$$

(5) Compute the ratio for selling external assets:

$$\theta_i(\sigma_i) = min\left[max\left(0, \frac{\sigma_i - \Delta b_i - \Delta c_i}{\alpha a_i}\right), 1\right];$$

where $\alpha \leq 1$ is the price of liquidation for a unit of external assets.

(6) Compute the inflow of liquidity for a bank: the paid cash plus withdraws from debtor banks plus liquidated external assets

$$\epsilon_i = \Delta c_i + \theta_i \alpha a_i + \sum_j \eta_i w_{ji};$$

(7) The iteration process ceases once the financial system reaches a steady state. Technically speaking, if ϵ_i and σ_i change very minimally ($< \zeta$) over several steps for all banks, then the iteration ceases; otherwise, start again from step (2).

After the iteration, banks that cannot meet outflows or needs of liquidity ($\epsilon_i < \sigma_i$) will default. By contrast, banks whose inflows of liquidity are sufficient to cope with needs of liquidity ($\epsilon_i = \sigma_i$) will remain solvent. After the dynamic allocation of liquidity, the final call of liquidity $\sigma_i = \Delta h_i + \sum_j \eta_j w_{ij}$ is much larger than the initial trigger Δh_i because of the knock-on effect in the interbank market.

The proportion of bankruptcy is not sufficient to fully represent the systemic risk. Therefore, as in Lee (2013), we consider another index, the *shortage of liquidity*, for the entire system. The shortage of liquidity is defined as total needs of liquidity minus total provisions of liquidity after the system achieves flow balance.

$$l^s = \sum_{i=1}^{N} (\sigma_i - \epsilon_i) \tag{2}$$

This expression represents households' unrealized needs for withdrawals or bad debt. In other words, the shortage of liquidity is the excess needs of liquidity that cannot able to be absorbed by the entire banking system, distinguishing the relative severity of the same proportion of bankruptcy.

4 Interbank network diversification and systemic risk

For the following simulations, we initialize the financial system using the steps introduced in Sect. 2. We focus on the effects of diversification on an interbank network from two aspects: the span of diversification and how banks diversify in an interbank market. Specifically, for our model, these two aspects correspond to the average degree and the topology of an interbank network. Based on two criteria, the proportion of bankruptcy and the shortage of liquidity, we investigate the systemic risk from the dynamic allocation mechanism of liquidity introduced in Sect. 3.

4.1 Span of interbank assets diversification and the proportion of bankruptcy

In considering the span of diversification in interbank assets, we introduce the connection level, i.e., the average degree in the interbank network. When the average degree in the interbank network (denser interconnections) is larger, one bank's portfolio is more sufficiently diversified.

Shocks from withdrawals of deposit are uniformly distributed and are the source of liquidity crises. The plot of proportion of bankruptcy as a function of the average degree of the interbank network is shown in Fig. 1. Using the parameters in Table 2, $N = 500$, $\gamma_b = 0.2$, $\gamma_v = 0.2$, $\gamma_s = 0.2$, and $\alpha = 0.4$, we show the results for the average of 50 simulations in Sects. 4.1 and 4.2.

As the average degree increases, the change in the proportion of bankruptcy varies under different initial conditions of liquidity shock. When the initial negative shock is small, the proportion of bankruptcy decreases when the average degree increases. Under this condition, the interbank network functions as mutual insurance. Conversely, when the shock is fairly large, the proportion of bankruptcy increases when the average degree increases. The interbank network is a transmission channel for the liquidity crisis because of the "knock-on" effect on financial institutions. This result is consistent with Ladley (2013) and Acemoglu et al. (2013): the same features that increase the system's resilience in "normal times" may function as significant sources of systemic risk and instability during "crises"; this emphasizes the "robust-yet-fragile" nature of the financial system.

We also present the same plots for the proportion of bankruptcy at a fixed average degree and as a function of p_0 for a random network. From Fig. 2, we can observe that when the liquidity shock is large, the proportion of bankruptcy is larger for a high connection level. In this scenario, the interbank network functions as a contagion channel. By contrast, when the shock is small, the proportion of bankruptcy is smaller for a high connection level. In this scenario, the interbank network functions as mutual insurance. We have the same result for a deficit-centered scale-free network, a surplus-centered scale-free network, and an interbank network created by the gravity model. These results are shown in "Appendix A". For a deficit-centered scale-free network, the transition of the proportion of bankruptcy is very sharp compared to those of the other three types of networks.

In fact, the threshold for the large withdrawal rate of household p_0 can be estimated. For a random network, the probability that two banks are connected increases

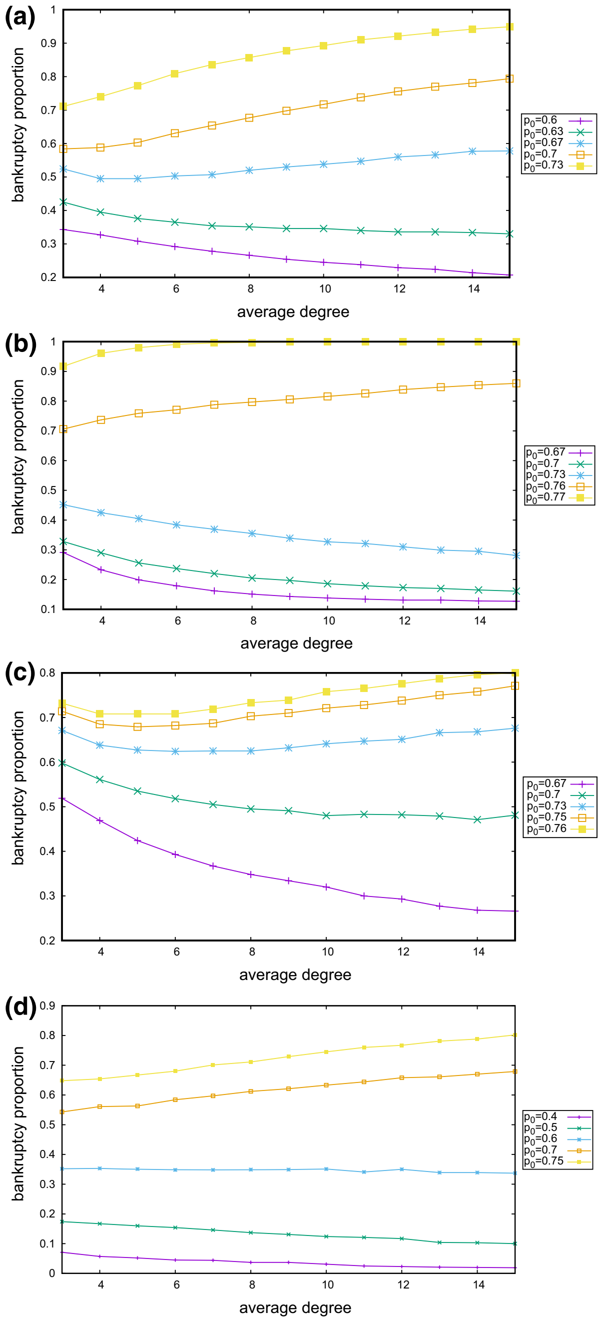

Fig. 1 The proportion of bankruptcy in interbank assets diversification. The x-axis is the average degree of the interbank network, the y-axis is the proportion of bankruptcy, and different curves represent different withdrawal rates of household p_0. The proportion of bankruptcy in **a** a random network, **b** a deficit-centered scale-free network, **c** a surplus-centered scale-free network, and **d** an interbank network generated by the gravity model

Table 2 Summary for parameters of the model

Parameter	Description	Value of parameter
N	Number of banks in the network	500
γ_b	Ratio of interbank loan	0.2
γ_v	Ratio of capital asset	0.2
γ_s	Ratio of deposit reserve	0.2
m	Average degree of interbank network	5
p_0	Rate of household withdrawal	0.7
α	Price of liquidation of a unit of external asset	0.4
ζ	Threshold of iteration termination	0.0001

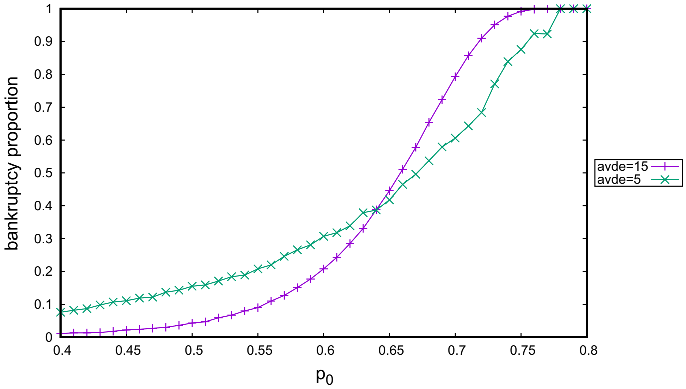

Fig. 2 The proportion of bankruptcy in interbank assets diversification. The x-axis is the withdrawal rate of household p_0, the y-axis is the proportion of bankruptcy, and the two curves represent interbank network degrees 5 and 15

as the average degree increases. In fact, a complete network is equal to a random graph with an Erdös-Rényi connection probability of 100%. Therefore, for banks in a complete interbank network, endogenously determined interbank liabilities d_i are equal to interbank assets b_i because of identity. Banks in a complete interbank network have the same total assets and interbank assets (total liabilities and interbank liabilities); in other words, they are homogeneous. Once one bank needs liquidity, all banks need liquidity from their interbank assets, and consequently, the interbank market will be cleared. Table 3 shows the balance sheet of a bank in a complete interbank network. To simplify the notation, total assets for one bank are set to one unit; thus $V_i = 1$.

For a complete network, the threshold for a "large" withdrawal rate can be calculated as follows:

$$p_0 h + d = \alpha a + b + c \tag{3}$$

From the parameters shown in Table 2, $\gamma_b = 0.2$, $\gamma_v = 0.2$, $\gamma_s = 0.2$, $\alpha = 0.4$, and according to Eq. (3), the threshold of the rate of withdraw $p_0^* = 0.65$. When $p_0 > p_0^*$, all banks will default in a complete network. However, when $p_0 < p_0^*$, no bank will

Table 3 The bank balance sheet for a complete interbank network

Assets	Liabilities
Cash	Households deposits
$c = \gamma_s(1 - \gamma_b - \gamma_v)$	$h = 1 - \gamma_b - \gamma_v$
Interbank assets	Interbank liabilities
$b = \gamma_b$	$d = \gamma_b$
External assets	Capital
$a = 1 - \gamma_s(1 - \gamma_b - \gamma_v) - \gamma_b$	$v = \gamma_v$

default. For a random network the threshold is approximated to p_0^* more closely than a scale-free network because banks are more heterogeneous in a scale-free network.

Given the household withdrawal ratio p_0 and the average degree, the proportion of bankruptcy increases for large asset depreciation (low liquidation price) and decreases for small asset depreciation (high liquidation price). The threshold for "large" asset depreciation can also be estimated in this manner; for the given household withdrawal ratio $p_0 = 0.7$, the threshold for price of liquidation $\alpha^* = 0.44$. The corresponding results are shown in "Appendix B".

4.2 Span of interbank assets diversification and the shortage of liquidity

We plot the shortage of liquidity as a function of the average degree of the interbank network as Fig. 3.

Whether the liquidity shocks from initial withdrawals of deposit are large or small, as the average degree increases, the shortage of liquidity in a random network, a surplus-centered scale-free network and an interbank network created by the gravity model will decrease, whereas the shortage of liquidity in a deficit-centered scale-free network will increase. As Lee (2013) shows, a core-periphery network with a deficit money centered bank shows the highest level of systemic shortage of liquidity, whereas a well-matched complete network has the lowest level. Therefore, as the average degree increases, the shortage of liquidity in a random network will decrease as it more closely approximates a complete network. Furthermore, with the average degree increment, the shortage of liquidity in a deficit-centered scale-free network will increase because hub banks have more interbank liabilities and are deficits in interbank markets, whereas the shortage of liquidity in a surplus-centered scale-free network will decrease because hub banks have more interbank assets and are in surplus. For the interbank network created by the gravity model, the function of the probability of links is $p_{ij} = p(V_i, V_j) \propto V_i V_j$. As the average degree increases, the interbank network becomes more approximate to a complete network, so the shortage of liquidity declines.

We have the same plot for the shortage of liquidity as a function of p_0 at a fixed average degree. For a deficit-centered scale-free network, the shortage of liquidity is higher for a high connection level of interbank network. For the random network, surplus-centered and interbank network created by the gravity model, the shortage of liquidity is lower for a high connection level of interbank network (Fig. 4).

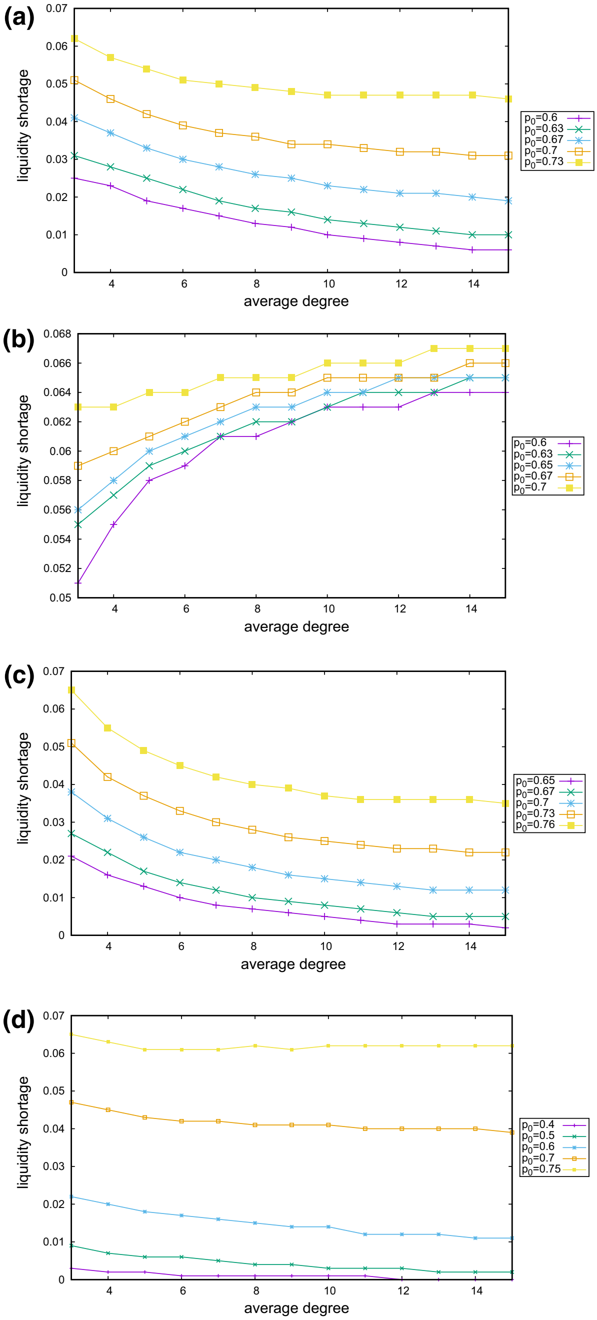

Fig. 3 The shortage of liquidity in interbank assets diversification. The x-axis is the average degree of the interbank network, the y-axis is the shortage of liquidity, and the different curves represent the different withdrawal rates, denoted as p_0. The shortage of liquidity in **a** a random network, **b** a deficit-centered scale-free network, **c** a surplus-centered scale-free network, and **d** an interbank network created by the gravity model

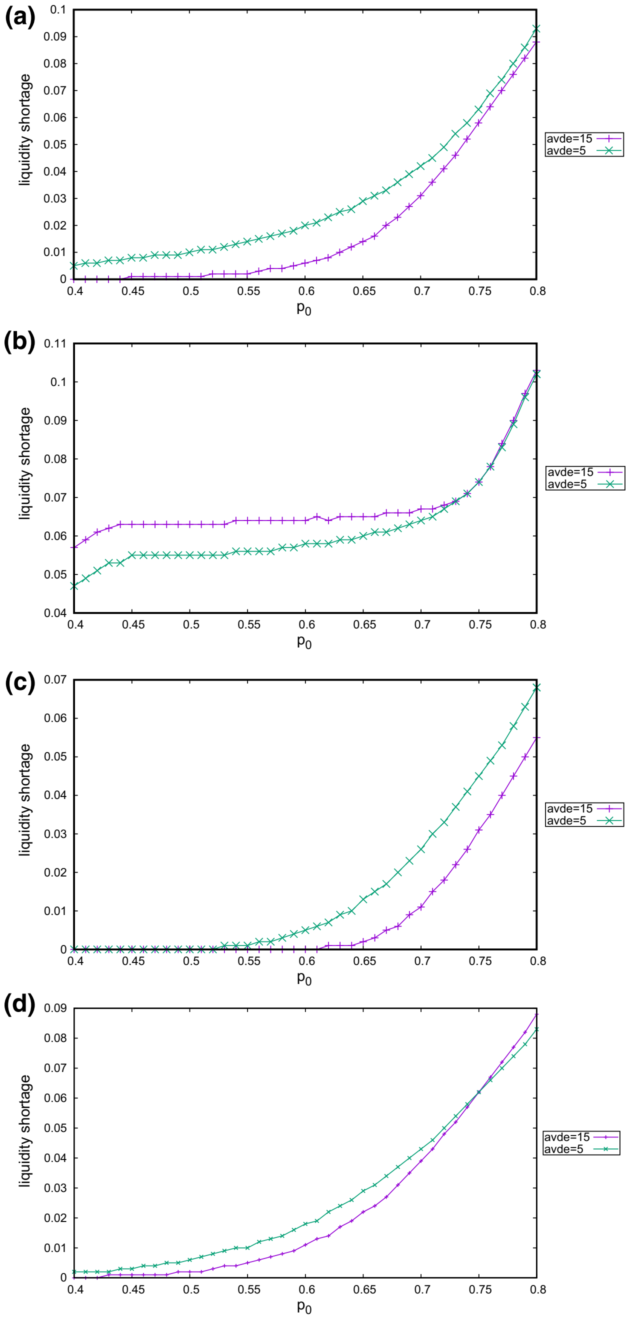

Fig. 4 The shortage of liquidity in interbank assets diversification. The x-axis is the household withdrawal rate p_0, the y-axis is the shortage of liquidity, and the two curves represent interbank network degrees of 5 and 15. The shortage of liquidity in **a** a random network, **b** a deficit-centered scale-free network, **c** a surplus-centered scale-free network, and **d** an interbank network created by the gravity model

4.3 Interbank topology and systemic risk

The results for the span of interbank asset diversification can also be interpreted as follows: liquidity shocks are transmitted through the interbank network. For liquidity concerns, in a random network, liquidity needs are shared and absorbed by more banks. Linkages among interbanks act more as absorbers for shock. However, for the scale-free network, all liquidity shocks will be forwarded to these central hub banks. Linkages among interbanks act more as transmitters for shock. The net position of hub banks in an interbank market makes a substantial difference in this scenario. Hub banks are systemically important banks, and their default will result in large shortage of liquidity.

For a fixed withdrawal ratio of household p_0, changing the price of liquidation of a unit of external asset α, we obtain the same result: the shortage of liquidity in a random network, a surplus-centered scale-free network and an interbank network created by the gravity model will decrease, whereas the shortage of liquidity in a deficit-centered scale-free network will increase. This result is shown in "Appendix B".

Among these four networks types, the surplus-centered scale-free network is better. First, as the initial shock increases, the changes of the proportion of bankruptcy remains highly stable. Second, the shortage of liquidity is the lowest of the four because hub banks are in surplus in the interbank market and provide liquidity for the entire financial system. We plot the proportion of bankruptcy and shortage of liquidity under a small, moderate and large initial withdrawal of household ($p_0 = 0.6, 0.7$ and 0.75). Based on parameters shown in Table 2, $N = 500$, $m = 5$, $\gamma_b = 0.2$, $\gamma_v = 0.2$, $\gamma_s = 0.2$, and $\alpha = 0.4$, the following results are the average of 50 simulations (Figs. 5, 6, 12).

When $p_0 = 0.6$ ($p_0 = 0.75$), as the interbank degree increases, the proportion of bankruptcy decreases (increases) for the four types of interbank networks. Thus, $p_0 = 0.6$ is a small shock while $p_0 = 0.75$ is a large shock. By contrast, $p_0 = 0.7$ is a moderate shock, and this result is shown in "Appendix C".

From these figures and our analysis, we can conclude that the shortage of liquidity in the surplus-centered scale-free network is less than that in the scale-free network generated by the gravity model, which, in turn, is less than that of the deficit-centered scale-free network. From the criterion of the shortage of liquidity, the surplus-centered scale-free network is better.

5 External assets diversification and systemic risk

According to the Modern Portfolio Theory (MPT) introduced by Markowitz (1952), an investor can reduce portfolio risk by holding combinations of instruments that are not correlated. In fact, in financial systems, different banks may hold the same types of external assets. Once one bank encounters a fire sale, the assets owned by that bank will depreciate, leading to the loss of the other banks' assets. This strategy of asset holding creates another channel for contagion; therefore, we introduce a bank–asset coupling matrix $L^A = \{a_{il}\}$. There are $M = 100$ types of assets in the market; each bank holds m_0 types of external assets. Thus, the probability of bank–asset connection

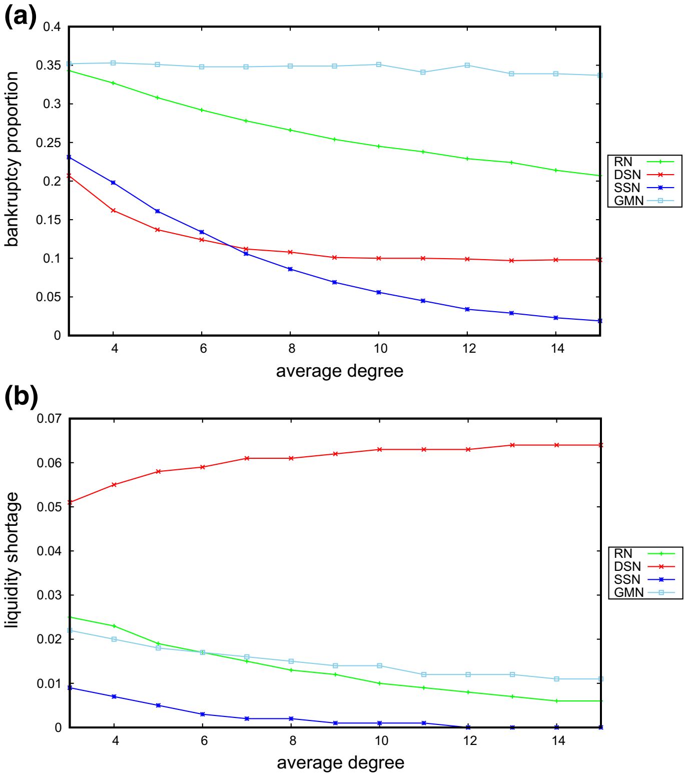

Fig. 5 Systemic risk for different interbank structures. The x-axis is the average degree of the interbank network. The y-axis in (**a**) is the proportion of bankruptcy and that in (**b**) is the shortage of liquidity. The four different curves represent the random network (RN), deficit-centered scale-free network (DSN), surplus-centered scale-free network (SSN), and interbank network created by the gravity model (GMN). The initial household withdrawal p_0 is 0.6

pba is m_0/M. The value of external assets for one bank a_i is evenly distributed across the assets that it owns.

The depreciation in the price of external assets is the actual reason for bank default in this financial system. If the price of external assets does not depreciate after dynamic allocation of liquidity, one bank can always obtain sufficient liquidity given the identity of assets and liabilities. Therefore, price fluctuations of external asset are of crucial importance to the financial system. In accordance with Schnabel and Shin (2004) and Cifuentes et al. (2005), we assume that the price of external assets is endogenously given by $\alpha(l) = exp(-\alpha_1 x_l)$, where x_l is the fraction of assets that have been liquidated and parameter α_1 is the index of market illiquidity. Note that $\alpha(l)$ decreases as x_l increases, which means that as more external assets are sold, the external assets will depreciate more. This creates a positive feedback effect.

The dynamic allocation of liquidity remains the same; the shock of liquidity is first absorbed by cash c_i, then by interbank assets b_i, and finally by external assets a_i. The difference here is that the price of asset endogenously depends on the percentage

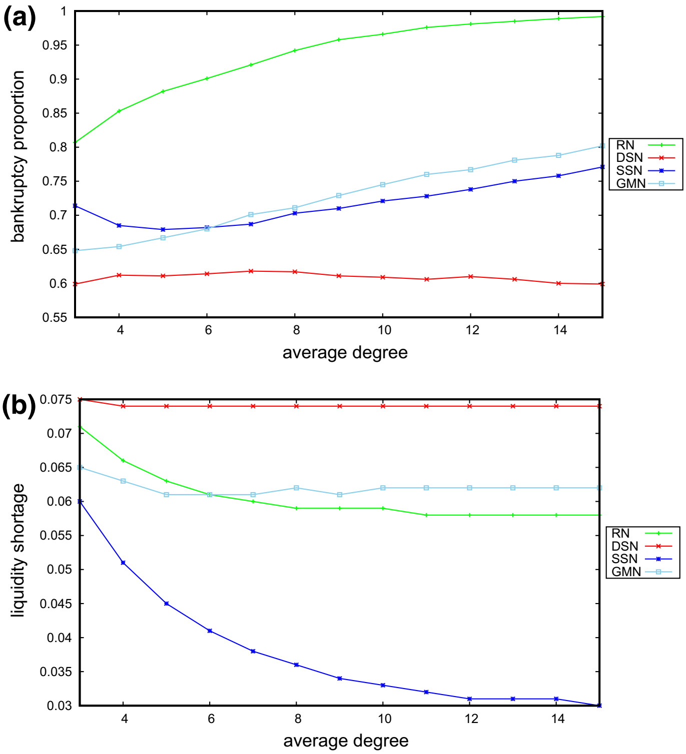

Fig. 6 Systemic risk for different interbank structures. The x-axis is the average degree of the interbank network. The y-axis in (**a**) is the proportion of bankruptcy and that in (**b**) is the shortage of liquidity. The four different curves represent the random network (RN), deficit-centered scale-free network (DSN), surplus-centered scale-free network (SSN), and interbank network created by the gravity model (GMN). The initial household withdrawal p_0 is 0.75

of assets sold in the market, and the price of asset decreases throughout the liquidity process.

Similar to Arinaminpathy et al. (2012), we also define the confidence for each bank and for the entire financial system. The state of confidence for one bank i is

$$s_i = s_i(\gamma_{vi}) = max(0, \gamma_{vi}/\gamma_v),$$

where γ_{vi} is the current ratio of capital asset and γ_v is the initial ratio of capital asset. The state of confidence for the entire financial system is

$$S = S(z_1, z_2) = 0.5(z_1 + z_2),$$

where $z_1 = \sum_{il} a_{il} / \sum_{il} a_{il}^0$ is the ratio of current total external assets to initial total external assets and $z_2 = \sum_i b_i / \sum_i b_i^0$ is the ratio of current total interbank assets to initial total interbank assets. The value of withdrawal from household is based on the state of confidence $\Delta h_i = \bar{\varepsilon}_i h_i$, where $\bar{\varepsilon}_i = max[0, min(\varepsilon_i, 1)]$, $\varepsilon_i \in N(\mu_i, \sigma)$

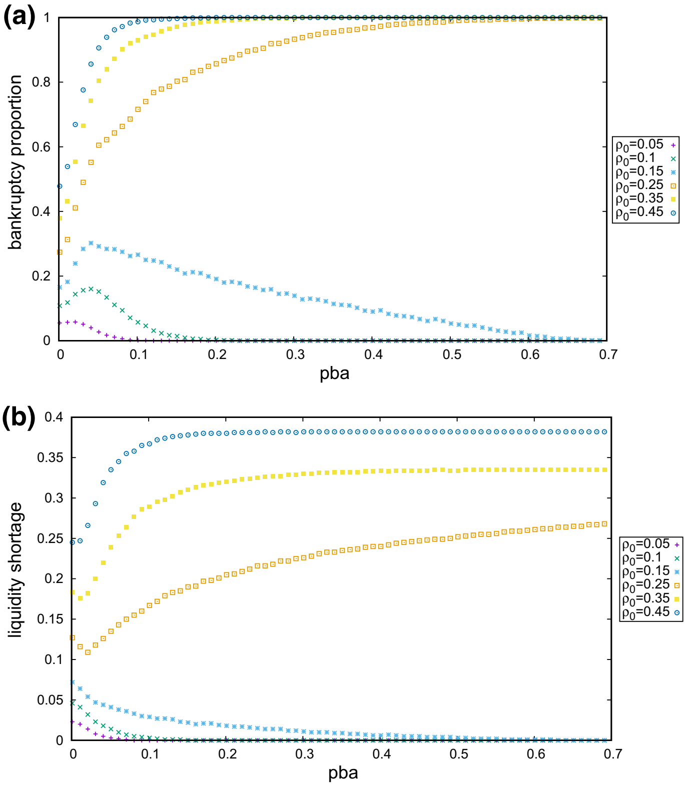

Fig. 7 Systemic risk in external assets diversification. The x-axis is the bank–asset connection probability pba. The different curves represent the different rates of initially shocked external assets ρ_0. **a** The proportion of bankruptcy is at the top, and **b** the shortage of liquidity is at the bottom

is the household withdrawal proportion, and it is based on the state of confidence $\mu_i(s_i, S) = 1 - s_i S$. We incorporate the endogenously determined price of external assets with the dynamic allocation mechanism of liquidity. During certain shocks of liquidity, the price for external assets $\alpha(l)$ is dynamic according to the proportion that has been liquidated. This algorithm is slightly different and is shown in "Appendix D".

Initially, ρ_0 of external assets are depreciated to zero, which will stimulate withdrawals from household. The interbank network is a random network with $N = 500$ and an average degree of $m_0 = 5$. Other parameters, i.e., γ_b, γ_v, and γ_s are in accordance with Table 2. Depreciated external assets are initially randomly selected, and the results are the average of 50 simulations. The systemic risk of the financial system is shown in Fig. 7.

We can observe that as the probability of bank–asset connection increases, the change in the proportion of bankruptcy varies under different initial conditions of liquidity shock. When the initial shock of liquidity is small, the proportion of bankruptcy and shortage of liquidity decrease as the probability of bank–asset con-

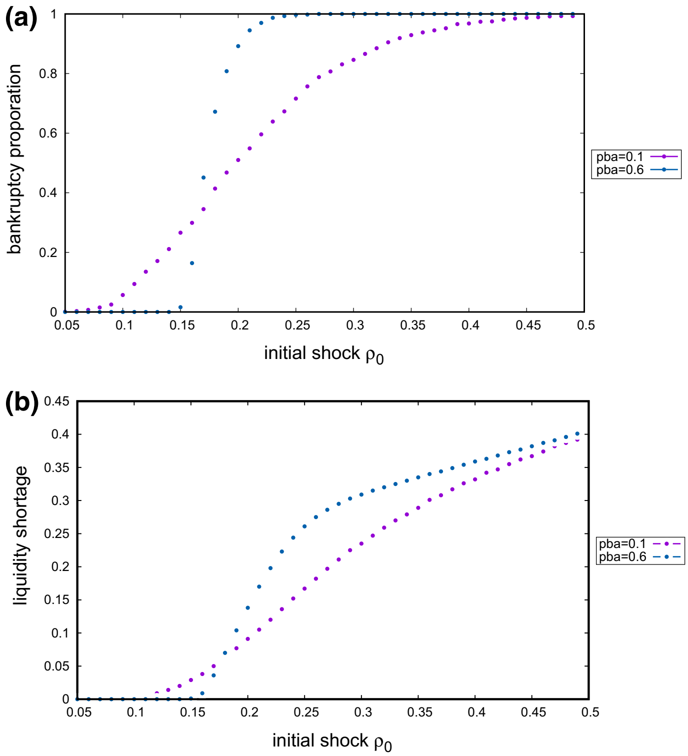

Fig. 8 Systemic risk in external assets diversification. The x-axis is initial shocked external assets ρ_0. The y-axis at the top is **a** the proportion of bankruptcy, and the y-axis at the bottom is **b** the shortage of liquidity. The two curves represent probability of bank–asset connection pba $= 0.1$ and pba $= 0.6$

nection increases. However, when the initial shock of liquidity is sufficiently large, the proportion of bankruptcy and shortage of liquidity increase, as one bank holds more types of external assets. Diversification actually involves a trade-off between individual and systemic risk, as noted by Wagner (2011) and Caccioli et al. (2014). When the initial shock of liquidity is small, by diversifying its portfolio, a bank can evenly liquidate its external assets when needed. Because the prices of external asset are endogenously determined by the quantity of liquidated external assets, holding more types of external assets prevents the large depreciation of one specific asset. However, when the initial ratio of external asset depreciation, and the ensuing shocks are large, nearly all banks must sell external assets to meet a call of liquidity. Overlapping portfolios can be another channel for contagion. More diversified holding of assets cannot always guarantee less risk.

We present the same plot for the fixed probability of bank–asset connection but as a function of ρ_0. We can observe that when the initial shock is small ($\rho_0 = 0.15$), the proportion of bankruptcy and shortage of liquidity are small for pba $= 0.6$ compared to pba $= 0.1$.However, as the initial shock increases, the network with a high probability of bank–asset connection undergoes a sharper transition (Fig. 8).

As noted by Beale et al. (2011), by diversifying its risks, a bank lowers its own probability of failure. However, when many banks diversify their risks by analogous means, the probability of large systemic failures can increase. For the diversification of external assets in a scale-free interbank network, we obtain the same result.

6 Conclusion

We model the financial system as a complex network composed of households, banks and external assets. Additionally, we focus on the impacts of diversification of assets on systemic risk. Diversification of assets in the interbank network and the bank–asset network are considered. Transfer of liquidity is treated as a diffusion flow in this network to show the cyclical liquidation of assets and liabilities. Liquidity configures dynamically in the financial system until a steady state is reached.

Three key results are highlighted. First, we find that the interbank network has a dual effect. This network can serve as mutual insurance and as a transmission channel during a liquidity crisis. Systemic risk is shown to be a non-monotonic function of the average level of diversification. The system presents a "robust-yet-fragile" property. When the initial shock of liquidity is small, connectivity engenders robustness. Conversely, when the shock is sufficiently large, fragility prevails because of the knock-on effect. Second, this paper shows that the topology of interbank networks has a significant effect on systemic stability. Compared to the more homogeneous random network, liquidity shocks are transferred to hub banks in a scale-free network. Additionally, hub banks are systemically important banks because of their endogenously high interbank liabilities (assets). Finally, this paper shows that diversification in a bank–asset network has a similar effect. Diversification actually involves a trade-off between individual and systemic risk.

For policymakers, this paper provides a natural objection to increasing financial stability by intensifying the mutual linkages among financial institutions. Diversification in one's portfolio can backfire when the shocks are large. The paper also provides evidence that the structure of interbank network must be considered. Hub banks with intense connections can have a great influence on the financial system.

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Appendix A: Systemic risk given a fixed average degree

The proportion of bankruptcy at a fixed average degree as a function of p_0 for scale-free networks (Fig. 9).

For a deficit-centered scale-free network, the proportion of bankruptcy transition is very sharp compared to those of the other three types of networks.

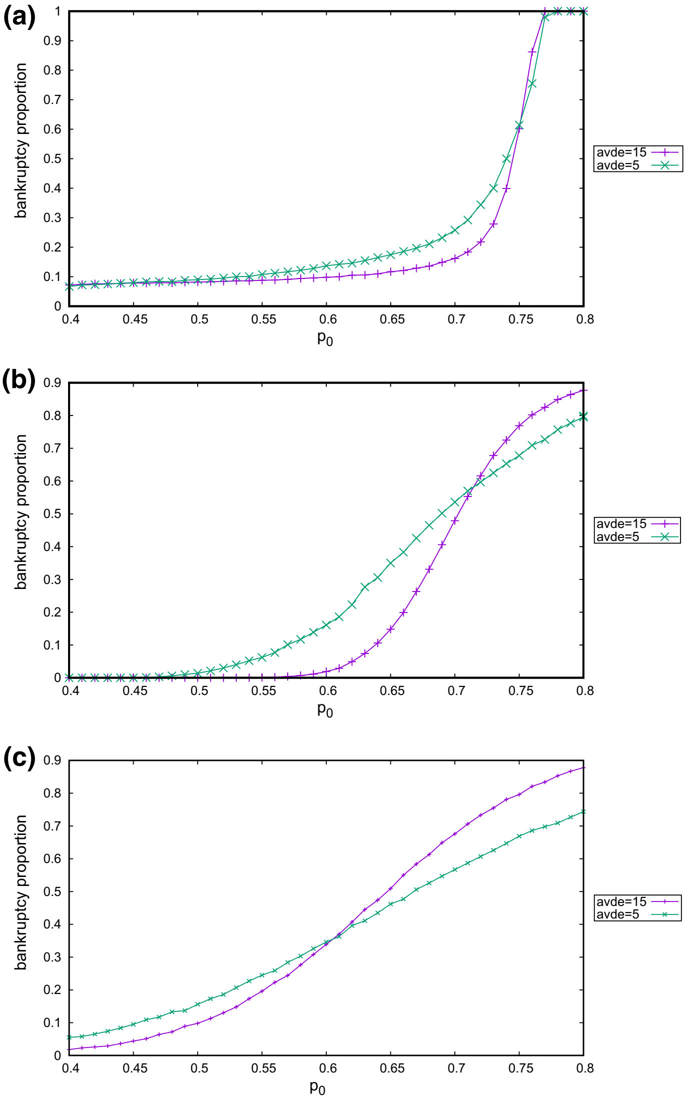

Fig. 9 The proportion of bankruptcy in interbank assets diversification. The x-axis is the household withdrawal rates p_0, the y-axis is the proportion of bankruptcy, and the two curves represent interbank network degrees of 5 and 15. The proportion of bankruptcy in **a** a deficit-centered scale-free network, **b** a surplus-centered scale-free network, and **c** an interbank network created by the gravity model

Appendix B: Systemic risk given a household withdrawal ratio

Consider the proportion of bankruptcy for a given household withdrawal ratio p_0 as the average increases; the proportion of bankruptcy increases under significant asset depreciation and decreases under small asset depreciation. The threshold can also

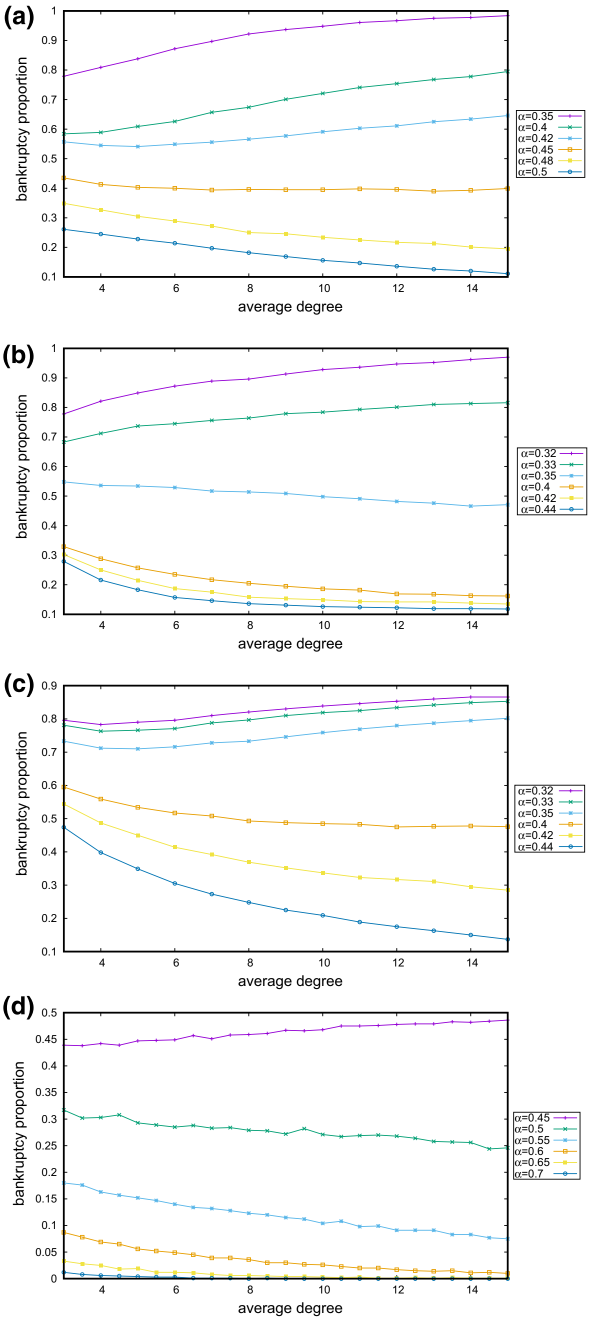

Fig. 10 The proportion of bankruptcy in interbank assets diversification. The x-axis is the average degree of the interbank network, the y-axis is the proportion of bankruptcy, and a different curve represents a different external asset liquidation price α. The proportion of bankruptcy in **a** a random network, **b** a deficit-centered scale-free network, **c** a surplus-centered scale-free network, and **d** an interbank network created by the gravity model. For the household withdrawal ratio $p_0 = 0.7$, the threshold is $\alpha^* = 0.44$

be estimated; for a given household withdrawal level of $p_0 = 0.7$, the threshold is $\alpha^* = 0.44$. The corresponding result is shown in Fig. 10, and the threshold for a random network is more approximate to α^*.

Considering the shortage of liquidity for a given household withdrawal ratio p_0 and after changing the liquidation price of a unit external asset α, we obtain the same result. The shortage of liquidity in a random network, a surplus-centered scale-free network, and an interbank network created by the gravity model will decrease whereas the shortage of liquidity of a deficit-centered scale-free network will increase as the average degree increases. The results are shown in Fig. 11.

Appendix C: Interbank topology and systemic risk

A comparison of the proportion of bankruptcy and the shortage of liquidity for four types of interbank networks is shown in Fig. 12.

When p_0 is 0.7, this is a large shock for the random and scale-free network generated by the gravity model. By contrast, this shock is small in the surplus-centered and deficit-centered scale-free interbank networks. The shortage of liquidity of the surplus-centered scale-free interbank network is the lowest of the four.

Appendix D: Dynamic allocation mechanism of liquidity incorporating the changing prices of external assets

The process for a dynamic transfer mechanism of liquidity that incorporates the bank–asset coupling matrix and dynamic price of external assets is as follows:

(1) For the first round set the outflow of liquidity for a bank i as $\sigma_i = \Delta h_i$;
(2) Compute the cash paid for the t iteration:

$$\Delta c_i(t) = min(c_i, \sigma_i(t));$$

(3) Compute the quota for the interbank withdrawals:

$$\eta_i(t) = min\left[max\left(\frac{\sigma_i(t) - \Delta c_i(t)}{b_i}, 0\right), 1\right];$$

(4) Update the outflow of liquidity for a bank: the initial withdrawals of household plus the withdrawals undertaken by creditor banks.

$$\sigma_i(t) = \Delta h_i + \sum_j \eta_j(t) w_{ij};$$

(5) Compute the portion for selling external assets:

$$\theta_i(t) = min\left[max\left(0, \frac{\sigma_i(t) - \Delta b_i(t) - \Delta c_i(t)}{\sum_l \alpha(t,l) a_{i,l}}\right), 1\right];$$

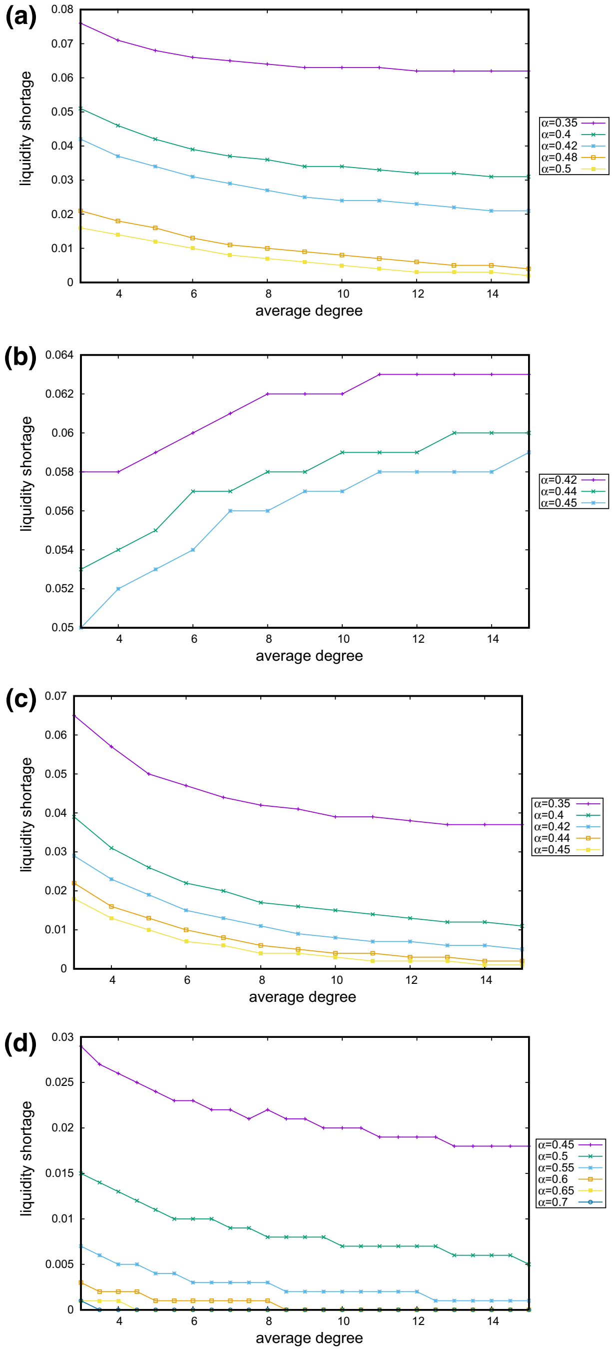

Fig. 11 The shortage of liquidity in interbank assets diversification. The x-axis is the average degree of the interbank network, the y-axis is the shortage of liquidity, and the different curves represent different external asset liquidation prices. The shortage of liquidity of **a** a random network, **b** a deficit-centered scale-free network, **c** a surplus-centered scale-free network, and **d** an interbank network created by the gravity model

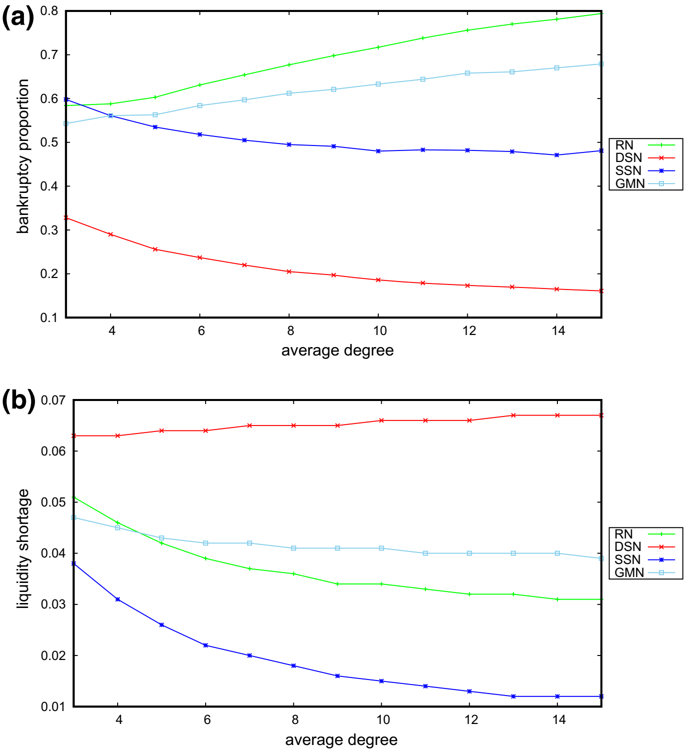

Fig. 12 Systemic risk for different interbank structures. The x-axis is the average degree in the interbank network. The y-axis in (**a**) is the proportion of bankruptcy, and that in (**b**) is the shortage of liquidity. The four different curves represent the random network (RN), deficit-centered scale-free network (DSN), surplus-centered scale-free network (SSN), and interbank network created by the gravity model (GMN). The initial household withdrawal value p_0 is 0.7

where $\alpha(t, l) = exp(-\alpha_1 x_l)$ is the liquidation value of external asset l for the t inner iteration. x_l is the fraction of the external asset l that has been liquidated, and α_1 is the illiquidity index for the market.

(6) Compute the inflow of liquidity for a bank: the cash paid plus withdraws from debtor banks plus liquidated external assets

$$\epsilon_i(t) = \Delta c_i(t) + \theta_i(t) \sum_l \alpha(t, l) a_{i,l} + \sum_j \eta_i(t) w_{ji};$$

(7) For all banks, when $\epsilon_i(t)$ and $\sigma_i(t)$ over several steps change very minimally ($< \zeta$), stop. Otherwise, start again from step 2, and enter the $t + 1$ inner iteration. The iteration process ceases after the financial system reaches a steady state.

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