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Is a co-jump in prices a sparse jump?

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ABSTRACT

Systematic co-jumps in asset prices are generally thought to account for only a small proportion of overall jumps. In actual observations, however, jumps in asset prices are often persistent, and the time of persistence varies. In this context, we develop a new rule to identify co-jumps and improve traditional tests by considering different sampling frequencies and different sampling starting points to re-evaluate the occurrence rate of systematic co-jumps in financial assets. We conduct a simulation experiment to show that the current test procedures generally underestimate the number of co-jumps when considering persistence, but that the proposed procedure can identify co-jumps more accurately. We also perform an empirical analysis using price data from the Shanghai 50 Index and its 25 constituent stocks in China's stock market. The average proportion of systematic co-jumps detected by the improved s-BNS is approximately 30%, which shows that the co-jump and even the systematic co-jump are not sparse jumps. The results also reveal the shortcomings of traditional jump tests in estimating persistent jumps and demonstrate that the proposed method can better detect the possible nondiversifiable risks between market indices and their constituent stocks, thereby contributing to financial risk management.

1. Introduction

Financial prices are usually characterized as a combination of continuous and discontinuous processes, where the discontinuous part generally refers to jumps caused by sudden changes in prices (Andersen et al., 2007; Back, 1991; Merton, 1976; Press, 1967). Das and Uppal (2004) defined the infrequent discontinuous process that occurs across multiple assets simultaneously as a cojump (similar statements about co-jumps can be found in Dungey et al., 2009; Dungey & Hvozdyk, 2012; Lahaye et al., 2011). Since co-jumps are closely related to financial market crashes and other systemic risks, accurately detecting co-jumps is of great significance in preventing financial risks. Gilder et al. (2014) defined the co-jumps between a stock and the market portfolio as systematic co-jumps. Since the stock index is essentially a portfolio of its constituent stocks, co-jumps that occur between a constituent stock and the index are referred to as systematic co-jumps in this study. These systematic jumps reflect the synchronization of risk between individual stocks and the index, which is an important indicator that investors need to consider when making decisions. The existing tests of co-jumps are mainly divided into two steps. The first step involves using techniques such as the BNS method (Barndorff-Nielsen, 2004; Barndorff-Nielsen & Shephard, 2006), ABD test (Andersen et al., 2007), L-M test (Lee & Mykland, 2008), s-BNS method (Andersen et al., 2010) and TOD method (Bollerslev et al., 2013) to identify jumps in individual asset or stock prices. The second step is to determine whether the identified jumps in different assets are co-jumps or just heterogeneous jumps resulting from the coexceedance rule proposed by Gilder et al. (2014) or some other derivative rules (Clements & Liao, 2017). Specifically, the traditional method to define a co-jump can be

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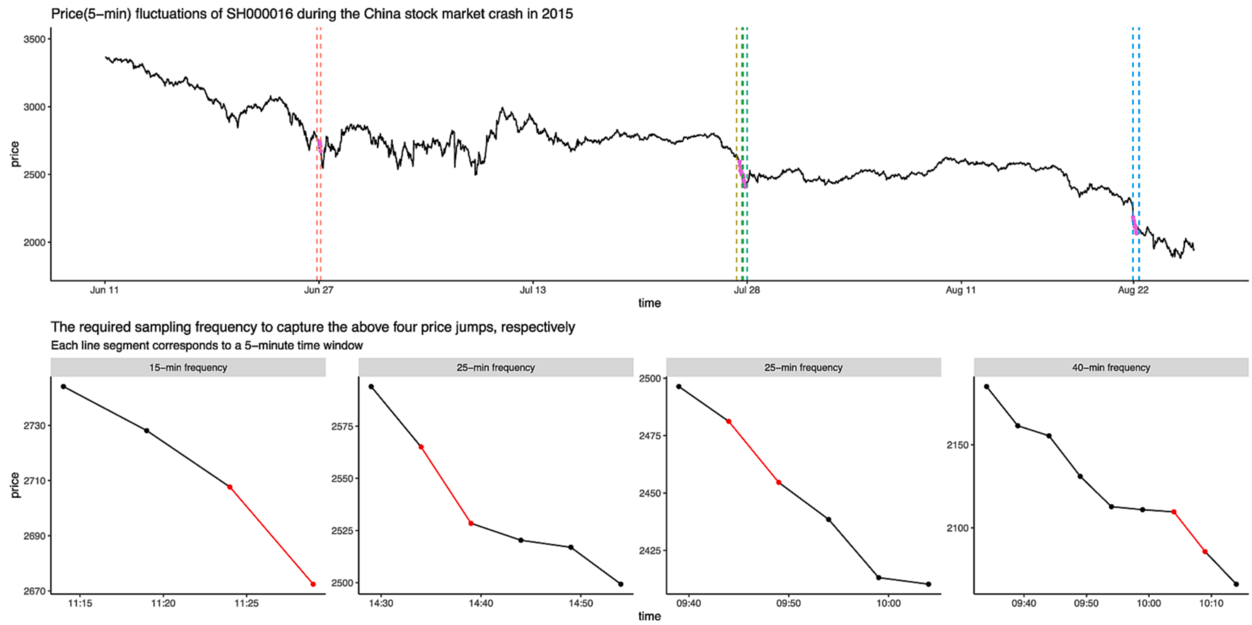

Fig. 1. The time of persistence of four jumps. The first row shows the 5-minute price series of the SSE 50 during the stock market crash, and the dotted line marks four persistent jumps that occurred during Jun 27, Jul 28 and Aug 22 in 2014. The second row zooms in on the complete process of these four price drops, which can more intuitively show that the time windows of these four persistent jumps are 15 min, 25 min, 25 min, and 40 min, respectively. To capture these jumps completely, the required sampling frequencies are 15 min, 25 min, 25 min, and 40 min. The red line marks price change that might also be detected as a jump in this subprocess at a sampling frequency of 5 min, but these price changes fail to reflect the persistent risk faced by the asset. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

summarized as follows: when the prices of two or more assets change drastically within the same observation interval, it is referred to as a co-jump. The implicit assumption is that co-jumps occur synchronously or at least approximately synchronously over a small observation interval. The traditional methods generally take the starting moments of these jumps as the basis for judgment.

However, we found that due to the impact of the same emergency, most financial price series will undergo persistent jumps, and the time of persistence for these jumps usually varies. We take the Shanghai 50 Index (SH000016) during the period from June to August 2015, when the stock crash occurred in China's market, as an example. The first row of Fig. 1 visualizes the 5-minute price series of the Shanghai 50 Index from 15:00 on June 11 to 15:00 on August 26 and marks four price processes that show a cliff-shaped decline over a short time span. We can clearly see that the several sudden changes in prices are noninstantaneous. The second row zooms in to show the complete process of these four price jumps, each of which has a different time window of persistence. If a fixed sampling frequency is used, then the four jump processes cannot be completely captured. We mark the intervals within each jump process that could be considered a jump at a sampling frequency of 5 min, and none of these intervals can fully reflect the persistent risk faced by the asset. This means that there are certain disadvantages to using a fixed sampling frequency to detect jumps with persistence, and the determination of persistence may rely on the frequency of sampling. If the persistence of jumps is ignored when there exists a certain time lag between these jumps, the number of co-jumps detected by traditional rules will be underestimated to a large extent, thus underestimating the level of systematic risk.

In general, the existing empirical research on co-jumps shows that the proportion of systematic co-jumps is relatively low, while the occurrence of heterogeneous jumps accounts for the majority of all jumps. Gilder et al. (2014) found that less than 18% of jumps across underlying stocks and market portfolios were systematic co-jumps. Wang et al. (2015) detected intraday co-jumps in China's spot and futures markets and found that only approximately one-third of these detected jumps were co-jumps. Clements and Liao (2017) believed that a co-jump only occurs when more than half of the 30 constituent stocks jump at the same time. The empirical results show that on average, approximately 4.1% of the intraday interval contains daily co-jumps. Arouri et al. (2019) identified the intraday jumps of three international exchange-traded funds and found that at the daily level, the proportion of co-jumps in each pair of the three funds is less than 30%. However, this conclusion positing the low frequency of co-jumps may not be robust. Russell and Engle (1998) pointed out that for discrete data that do not arrive within equal time intervals, such as a financial price series, certain errors will be generated if a fixed sampling frequency and starting point are used for analysis. When estimating price volatility, considering that price fluctuations do not regularly occur within a fixed interval, traditional methods also result in bias. Cho and Frees (1988) and Gerhard and Hautsch (2002) both proposed improved price volatility estimators. Similarly, as mentioned above, traditional jump tests based on high-frequency data are usually constructed under an instantaneous jump assumption has been made, which means that they are greatly affected by the sampling frequency and sampling starting point (Andor & Bohák, 2017). Motivated by our observations, we firmly believe that the techniques for identifying co-jumps should be improved by considering the persistence of jumps and sampling these jumps with different frequencies and start points.

In this article, we redefine the rules for classifying a co-jump, which allow for jumps that do not occur at the same time and have different levels of persistence to still be detected as co-jumps under certain conditions. This definition better reflects the actual dynamics of the financial market, thus reducing the possibility of underestimating systematic risks. Andor and Bohák (2017) proposed the multisample BNS method (also called multisample BPV) and found that it could increase the event (persistent jump) detection rate by a factor of three compared with traditional BNS methods. Inspired by Andor and Bohák (2017), we first reveal the limitations of the traditional jump tests by changing the sampling starting point and sampling frequency. Then, we improve the s-BNS, LM and TOD methods by considering jump persistence at the intraday level in jump detection. We also update the rule for identifying co-jumps and thus propose a complete procedure with which to test jumps and cojumps. The results show that the proposed scheme can detect more systematic co-jumps, which indicates that there may be a higher degree of risk synchronization between individual stocks and the market index. From the perspective of risk management, this nondiversifiable risk deserves more attention from investors since market-level news can result in more systematic shocks than was previously expected.

2. Methods

This study focuses on the test results of co-jumps, which first requires the identification of jumps in the price series of individual assets and then uses specific rules to determine which jumps occurring in multiple assets can be regarded as co-jumps. In this section, we first introduce several traditional tests for detecting intraday jumps, and then we propose an improved procedure for (co-)jump detection, which applies the method that expands the detection of traditional jumps to include persistent jumps and updates the rule for identifying co-jumps when jump persistence is considered.

2.1. BNS and s-BNS tests

The original BNS test method (Barndorff-Nielsen, 2004) assumes that the jump component in the price process is instantaneous, that is, a jump only has a transient impact on the price. Therefore, it is often characterized by a compound Poisson process. Correspondingly, the price $p(t)$ is a semimartingale process within the general jump-diffusion setting:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + J(t)dN(t), \tag{1}$$

where $\mu(t)$ and $\sigma(t)$ indicate the annual return and the covariance matrix of the price. dt is the unit time step relative to the year, and $J(t)$ is the jump component.$W(t)$ and $N(t)$ represent the Brown process and Poisson process, respectively.

Barndorff-Nielsen et al. (2004) considered the bipower variation measure of daily return, BV_{T}, as a consistent estimate of the population variance of the price process. Therefore, the contribution of jumps to population variance can be estimated as the difference between realized volatility and bipower variation, namely, $RV_{\mathrm{T}} - BV_{\mathrm{T}}$. We denote the corresponding statistics of jump as $Z_{BNS,\mathrm{T}}$ and $Z_{BNS,T} \xrightarrow{D} N(0,1)$. This can be defined as:

$$Z_{BNS,T} = \frac{1 - \frac{BV_{\mathrm{T}}}{RV_{\mathrm{T}}}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)M^{-1}\max\left(1, \frac{TP_T}{BV_{\mathrm{T}}^2}\right)}}, \tag{2}$$

where $V_T = \sum_{i=1}^{M} {r_{T_i}}^2, BV_{\mathrm{T}} = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M}|r_{T_i}||r_{T_{i-1}}|,\ TP_T = \mu_{\frac{4}{3}}^{-3}\left(\frac{M^2}{M-2}\right)\sum_{i=3}^{M}|r_{T_{i-2}}|^{\frac{4}{3}}|r_{T_{i-1}}|^{\frac{4}{3}}|r_{T_i}|^{\frac{4}{3}},\ \ \mu_{4/3} = 2^{\frac{2}{3}}\Gamma(7/6)\Gamma(1/2)^{-1}$.

Where i denotes the i^{th} high-frequency moment in day T, and there are M high-frequency returns within a day. The returns mentioned in this article default to logarithmic returns, which are defined as the difference between logarithmic prices, i.e., $r_{T_i} = logP_{T_i} - logP_{T_{i-1}}$, where P represents the asset price.

However, the method can only detect jumps at the daily level. S-BNS (Andersen et al., 2010) extends the BNS test to identify intraday jumps. This method indicates that after using the BNS test to determine the day on which the jump occurs, if i intraday jumps are detected by the relatively largest i returns during the day, the remaining $M-i$ returns on this day are scaled by $M/(M-i)$ to calculate the adjusted volatility and produce the updated BNS statistics. This procedure is repeated until the statistics no longer reject the null hypothesis that there is no jump.

2.2. L-M test

Lee and Mykland (2008) proposed a new method for detecting jumps as follows: when determining whether a jump has occurred at time T_i, the K observations before T_i are considered to estimate realized bipower variation as instantaneous volatility. The ratio of the return at T_i to the estimated instantaneous volatility is used as the L-M statistic and serves as the basis to check whether a jump occurred at T_i and if so, the size of the jump. If time T_i corresponds to the θth observation, the L-M statistic can be expressed as:

$$L(T_i) = \frac{logP_{T_i} - logP_{T_{i-1}}}{\widehat{\rho(T_i)}}, \tag{3}$$

where $\widehat{\rho(T_i)}^2 = \frac{1}{K-2}\sum_{j=\theta-K+2}^{\theta-1}|logP_{T_j} - logP_{T_{j-1}}||logP_{T_{j-1}} - logP_{T_{j-2}}|$.$K$ is a self-defined parameter. Lee and Mykland (2008) suggested that the value of K should be within the range of $\sqrt{252^*M}$ to 252^*M, where M is the number of high-frequency returns in a day. Assuming that there are φ trading days in a fixed time horizon, then the total number of observations will be $\mathscr{N} = \varphi^*M$. If $\mathscr{L}(T_i)$ satisfies $\frac{|\mathscr{L}(T_i)|-C_{\mathscr{N}}}{S_{\mathscr{N}}} > -\log(-\log(1-\alpha))$, where $C_{\mathscr{N}} = \frac{(2log\mathscr{N})^{\frac{1}{2}}}{c} - \frac{log\pi+\log(log\mathscr{N})}{2c(2log\mathscr{N})^{\frac{1}{2}}}$, $S_{\mathscr{N}} = \frac{1}{c(2log\mathscr{N})^{\frac{1}{2}}}$, $c = \sqrt{\frac{2}{\pi}}$ and α denote the confidence level, then the null hypothesis that there is no jump at time T_i is rejected.

2.3. TOD test

TOD_i measures the ratio of the diffusive variation of the ith ($i = 1,\cdots,M$) moment of the day relative to its average value for the day. The series of TOD,$\{TOD_i\}_1^M$, generally exhibits an intraday U-shaped structure as a function of i over trading day (Bollerslev et al., 2013). TOD_i is then used to adjust the criteria threshold to determine jumps of each time interval in a day, which can be expressed as:

$$TOD_i = \frac{M\sum_{T=1}^{\varphi}|r_{T_i}|^2 I\left(|r_{T_i}| \le \epsilon\sqrt{BV_T \wedge RV_T}\, M^{-\varpi}\right)}{\sum_{s=1}^{MT}|r_s|^2 I\left(|r_s| \le \epsilon\sqrt{BV_{\left[\frac{s}{M}\right]} \wedge RV_{\left[\frac{s}{M}\right]}}\, M^{-\varpi}\right)}, \tag{4}$$

where $\epsilon > 0$, $\varpi \in (0, 0.5)$ and $I(\cdot)$ are indicative functions. TOD_i measures the average of high-frequency returns at the i th moment in the day relative to daily returns. ϖ and ϵ are usually set to the empirical values of 0.49 and 2.5. Hence, the critical value for judging whether a jump occurs in the s th moment will be $\eta = \epsilon\sqrt{(BV_{[s/M]} \wedge RV_{[s/M]})^* TOD_{s-[s/M]M}}M^{-\varpi}$, $s = 1, 2, \cdots, MT$. If $|r_s| \ge \eta$, then a jump is considered to have occurred at this moment, and vice versa.

2.4. The improved procedure of (co-)jump detection

As mentioned earlier, all the above traditional tests are proposed based on the price model that views jumps as instantaneous components, which clearly contradicts our actual observation of jumps. Andor and Bohák (2017) proposed a multisample BNS method that considers the impact of sampling frequency and sampling starting point and redefined the critical value of the new statistics. This method calculates the BNS statistics under different sampling frequencies and sampling starting points for each day. If the maximum BNS statistic in a day is greater than the critical value, then a jump can be considered to have occurred on that day. This can precisely identify jumps that are not detected by traditional BNS due to the fixed sampling frequency and starting point. Thus far, we can see that the traditional BNS has room for improvement. However, the traditional L-M and TOD can hardly be improved in a similar way since the latter two detect intraday jumps rather than daily jumps. For consistency, we will introduce a unified approach to improving the procedure of jump detection and co-jump detection under traditional jump tests by accounting for jump persistence. Meanwhile, we provide a conceptual definition of the time of jump persistence and introduce a method for estimating the time of jump persistence that relies on the results of jump tests reported in this section.

Based on this phenomenon, we simply define the time of jump persistence as the time it takes for an individual jump to unfold, from initial occurrence to decay to the end. In light of multisample BNS, the key to not underestimating risk is to traverse every small time unit and determine whether a jump has occurred at this time point. Specifically, to avoid underestimating risks due to a the low sampling frequency of returns and to avoid market microstructure noise due to the high sampling frequency of returns, it is necessary to choose a minimum suitable sampling interval, τ^*. Under this frequency, the traditional tests referenced above are used to detect jumps. If jumps in the same direction occur continuously, they can be regarded as an individual jump with a persistent process . If the number of consecutive jumps in the same direction is g, then the time of jump persistence is $\tau = \tau^* g$. Hence, these approaches are referred to as improved s-BNS, improved L-M and improved TOD methods.

Since the identification of jumps has been improved, the rule for determining co-jumps should be updated accordingly. The traditional technique proposed by Gilder et al. (2014) to detect co-jumps is referred to as the coexceedance rule,

$$\sum_{j=1}^{N} I\left(Jump_{T_i,j} > 0\right)\begin{cases} \ge 2Cojump, \\ \le 1NoCojump, \end{cases} \tag{5}$$

where $Jump_{T_i,j}$ denotes the number of jumps at moment i on day T in the jth asset among N assets, indicating that co-jumps should occur within the same time interval; otherwise, they are heterogeneous jumps. However, we believe that if the time of persistence of several jumps with the same direction overlaps, a co-jump can be considered to have occurred. The updated rule determines whether a co-jump occurs by observing whether the time of persistence of jumps with the same direction in a jump series of any two assets overlap. We denote the arrival time of the qth jump in asset m on day T as T_q and denote the time of persistence of this jump as $\tau_{T,q,m}$. Accordingly, the arrival time of the q'th jump in asset n on day T is denoted as T_q, and the time of persistence of this jump is denoted by $\tau_{T,q',n}$. This rule can be expressed as

$$I\left(\{[T_q, T_q + \tau_{T_q,m}] \cap [T_{q'}, T_{q'} + \tau_{T,q',n}]\} \ne \varnothing\right)^* I\left(Sign_{T,q,m} = Sign_{T,q',n}\right)\begin{cases} = 1Cojump, \\ = 0NoCojump, \end{cases} \# \tag{6}$$

Table 1
Parameter setting of Monte Carlo simulation.

Parameter	Value
Annual return	10%
Annual volatility	20%
Initial price	$U(500, 1000)$
Probability of heterogeneous jumps	0.00025 for positive jumps 0.00025 for negative jumps
Probability of co-jumps	0.0005 for positive co-jumps 0.0005 for negative co-jumps
Size A of the initial jump component of the positive persistent jump	$N(5, 1)$
Size A' of the initial jump component of the negative persistent jump	$N(8, 1)$
The time of jump persistence τ	$U(10, 20)$
The decay process of the jump component	$Aexp(-t), t = 0, 1, 2, \cdots, \tau$ for the positive persistent jump $A'exp(-t), t = 0, 1, 2, \cdots, \tau$, for the negative persistent jump

where $Sign_{T,q,m}$ and $Sign_{T,q',n}$ indicate the sign of the jump, and $[a, b]$ represents the time range starting from a and ending at b.

3. Simulation analysis

We conduct a simple Monte Carlo simulation to illustrate the shortcomings of traditional tests that ignore jump persistence in detecting co-jumps. The model that includes jumps with persistence is also discussed in the work of Song and Li (2022), and we will refer to the price mechanism referenced in this paper for simulations. The stochastic differential equation form of the price model can be expressed as follows:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + J'(t)dN(t), \tag{7}$$

where $J'(t)$ represents the size of the jump component of a persistent jump, rather than an instantaneous jump, at moment t. Specifically, this price process can be considered to be a superposition of an ordinary Brownian motion process and a filtered Poisson process. The so-called filtered Poisson process defines each arrival event that obeys the Poisson distribution as not being instantaneous but rather having a certain persistence. That is, the jump component driven by the filtered Poisson process has a persistent impact on the price. Additionally, the degree of impact generally decays with time. Based on the dynamics of the price mentioned above, we apply a Monte Carlo simulation to generate the price series of two assets. We assume that the unit step of the series is 1 min and the number of trading days is 100. Since there are 240 trading minutes on each day in China's stock market, we generate 24,001 simulated prices and thus obtain 24,000 1-minute returns. Table 1 lists the parameters involved in the simulation and their range of values.

Neglecting the years with significant systematic risk, we obtain an average annual return of approximately 0.10 and an annual volatility of approximately 0.22 based on 8 years of historical price data of the SSE Composite Index ranging from 2009 to 2016. Therefore, we set the annual return to 0.1 and the annual volatility to 0.2 in the simulation. We refer to individual stock price data from the actual Chinese market, such as the constituents of the SSE 50, which is formally introduced in the empirical section, to determine

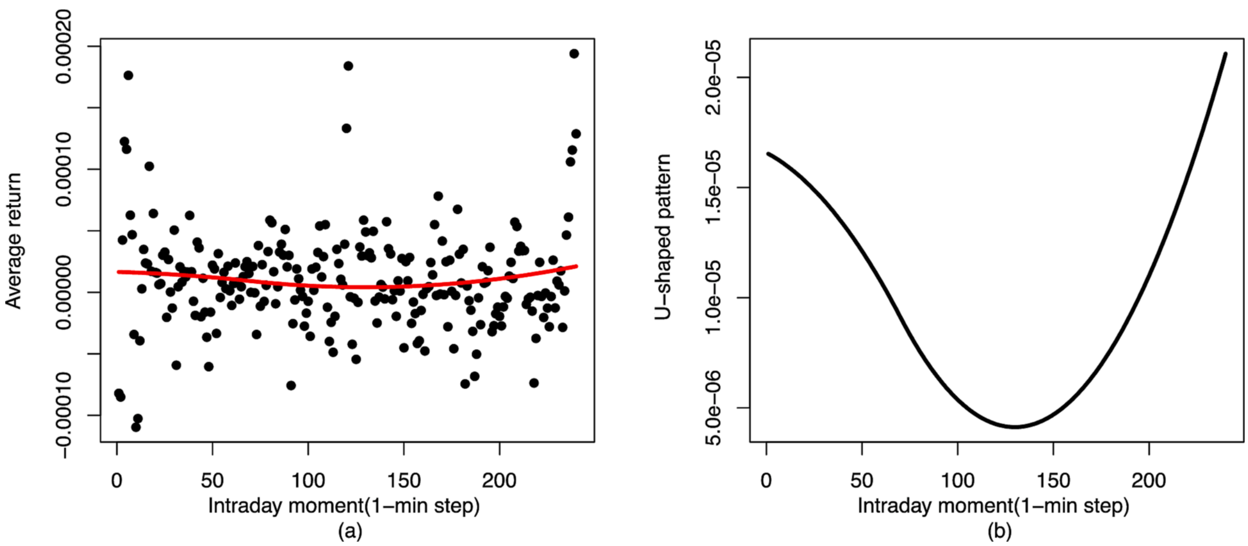

Fig. 2. The U-shaped pattern of intraday returns.

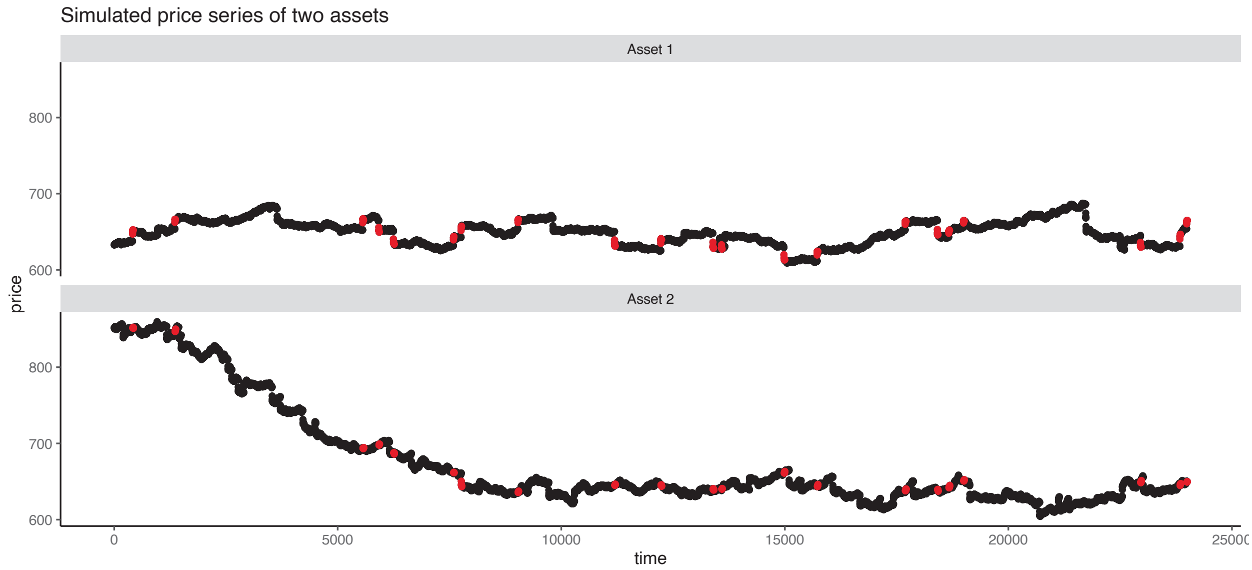

Fig. 3. The simulated price series of two assets. The co-jumps simulated in the two assets are marked with red dots, and the co-jump of asset 2 lags behind that of asset 1 by 5 time steps.

Table 2
Test results of co-jumps in simulated price series.

Method	Sampling frequency	The average number of real co-jumps	The average number of detected co-jumps	The average detection rate of co-jumps
s-BNS	5 min	25	9	35.54%
	10 min	25	4	15.80%
	20 min	25	2	7.97%
	improved	25	18	71.30%
TOD	5 min	25	7	27.40%
	10 min	25	6	23.33%
	20 min	25	4	15.35%
	improved	25	15	59.87%
L-M	5 min	25	3	11.66%
	10 min	25	4	15.55%
	20 min	25	2	7.71%
	improved	25	8	31.75%

Note: the average jump detection rate is not the ratio of the average number of detected jumps to the average number of real jumps.

parameters such as the initial price of stocks and the initial size of persistent jumps. To avoid the price being less than 0 due to random wandering during the simulation, the initial price should be set to a higher value. Considering that the stock prices of the constituent stocks fluctuate between a few Chinese Yuan and over a thousand Chinese Yuan, the initial prices of the two series are set to follow a uniform distribution on [500, 1000]. Campbell and Hentschel (1992), Bollerslev, Litvinova, and Tauchen (2006), and Dennis, Mayhew, and Stivers (2006) showed stronger volatility for negative returns than for positive returns. This asymmetric volatility of returns results in a higher average size of negative jumps than of positive jumps in the real market. In the simulation, for simplicity, we assume that the size of the initial jump component of the persistent jump in these two simulated price series obeys a normal distribution with the same parameters. The results of the traditional jump tests for each constituent stock in SSE 50 in 2014 also provide a hint as to the relative relationship between the size of jumps and prices. When using a 5-minute sampling frequency, the average size of the positive jumps is approximately 0.005 to 0.01 of the average price series, and the average size of the negative jumps is approximately 0.008 to 0.016 of the average price series. When the initial price ranges from 500 to 1000, the size A of the initial jump component of positive persistent jumps is set to follow a normal distribution with a mean of 5 and a standard deviation of 1. The size A' of the initial jump component of negative persistent jumps is set to follow a normal distribution with a mean of 8 and a standard deviation of 1. Under this setting, the mean values of 5 and 8 in the normal distribution ensure that the relative relationship between the initial size of the jump and the price is consistent with the actual situation. Setting a standard deviation of 1 ensures that there is only a near-zero probability that the size of the initial jump components will be less than 0 during the sampling process.

For the probability of occurrence of jumps, assuming that there is a 0.0005 probability of heterogeneous jumps in both assets, the probability of positive jumps and negative jumps is 0.00025. It is assumed that there are certain co-jumps in the two assets, and that the

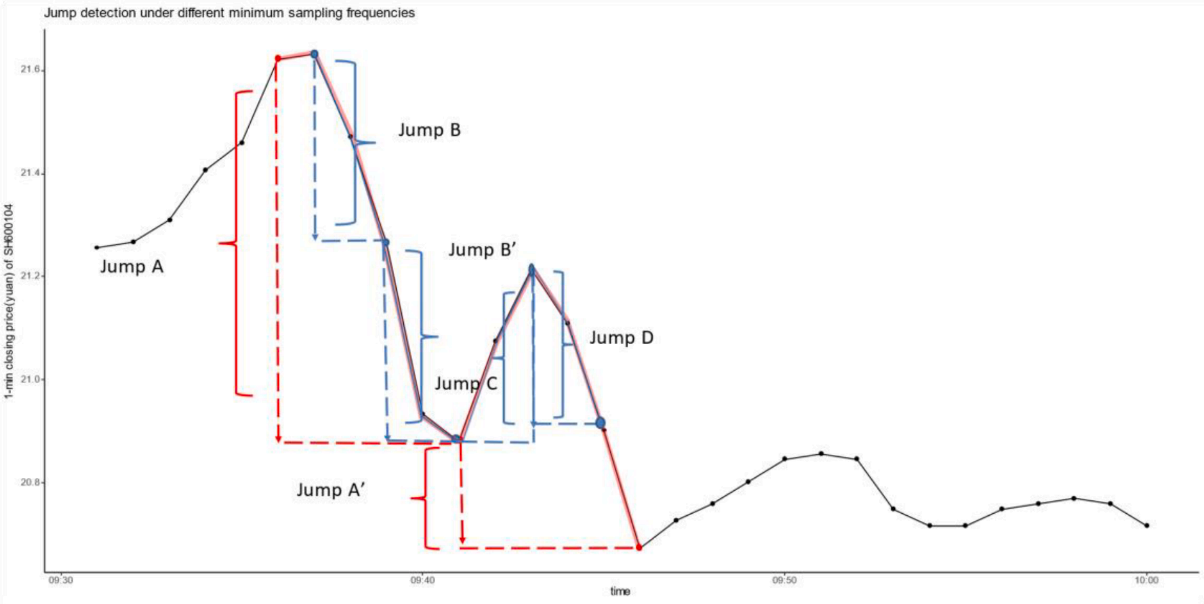

Fig. 4. Jump detection at 5-minute and 2-minute sampling frequencies for the partial data of sh600104 by s-BNS. The identified jumps at 2-minute and 5-minute sampling frequencies are marked in blue and red, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

probability of occurrence of positive and negative co-jumps are both 0.0005, which are set slightly higher than that of heterogeneous jumps to ensure an obvious result of the co-jump test. Therefore, the total jump intensity of a simulated stock is set to 0.0015, which is in line with that of a real stock. Meanwhile, both the heterogeneous jumps and the co-jumps are set to be persistent since such overlap may also be persistent when there is an overlap between the jump intervals of the two stocks. The decay process of jumps is assumed to obey a simple exponential process. Since we test at sampling frequencies of 5, 10, and 20 min and strive to avoid identifying consecutive jumps as discontinuous individual jumps due to a fast decay process, we make the time span of jump persistence obey a uniform distribution [10, 20]. Additionally, we intend to show that the test method proposed in this paper can detect co-jumps with inconsistent starting points, so we assume that there exists a lag of 5 min in the co-jumps that occur between the two stocks.

A peculiar intraday U-shaped pattern in returns has been found in many studies (Wood, McInish, & Ord 1985; Harris 1986). We calculate the average return corresponding to the 240 high-frequency moments of the day based on actual data from the SSE Composite Index in 2014 and use the U-shaped structure fitted to this average return series as the cyclical structure to be used in the simulation series. Subgraph (a) in Fig. 2 shows the fitted results, and subgraph (b) displays the extracted intraday U-shaped pattern from (a). Fig. 3 displays the price series of the two assets in one of the simulations with the co-jumps marked.

At sampling frequencies of 5 min, 10 min and 20 min, the traditional s-BNS test, TOD test and L-M test are used to test for co-jumps in the simulation series. Then, the improved s-BNS, as well as the improved TOD and L-M tests, are combined with the proposed updated rule to detect these co-jumps. These improved tests use 2 min as the minimum sampling frequency. We perform 10,000 simulations and list the average test results of each method on the co-jumps displayed in Table 2.

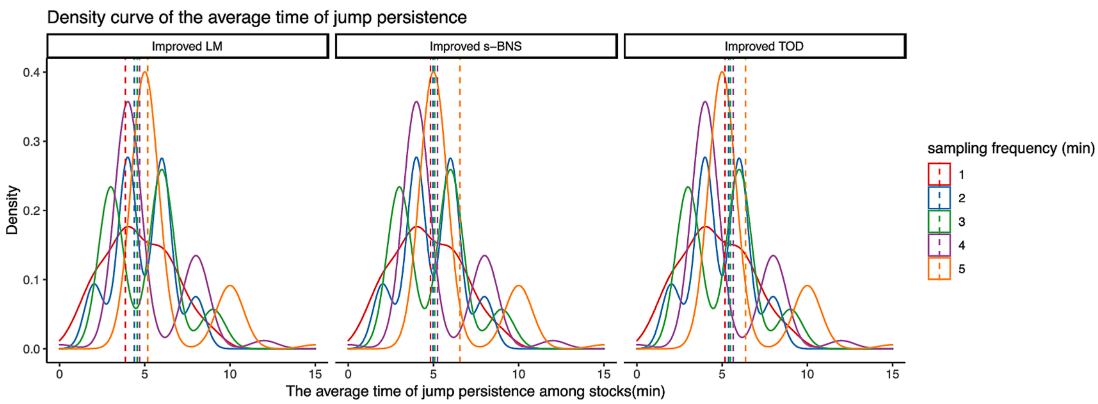

Fig. 5. Density plots of the average time of jump persistence.

Table 3
Cojump identified by BNS Test.

Stock	5 min			10 min			20 min		
	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps
SH000016	9			4			0		
SH600009	5	0	0.00%	2	0	0.00%	0	0	/
SH600016	7	1	14.29%	3	0	0.00%	0	0	/
SH600028	5	0	0.00%	7	0	0.00%	0	0	/
SH600030	3	0	0.00%	3	1	33.33%	0	0	/
SH600036	7	2	28.57%	1	0	0.00%	1	0	0.00%
SH600048	8	0	0.00%	4	1	25.00%	0	0	/
SH600050	9	1	11.11%	4	1	25.00%	0	0	/
SH600104	4	0	0.00%	5	1	20.00%	0	0	/
SH600519	9	1	11.11%	5	0	0.00%	0	0	/
SH601166	5	0	0.00%	3	1	33.33%	0	0	/
SH600029	12	2	16.67%	11	1	9.09%	0	0	/
SH600887	6	1	16.67%	5	0	0.00%	0	0	/
SH601688	12	1	8.33%	5	0	0.00%	1	0	0.00%
SH601901	14	2	14.29%	4	1	25.00%	1	0	0.00%
SH601169	9	0	0.00%	5	0	0.00%	0	0	/
SH601328	5	0	0.00%	2	0	0.00%	0	0	/
SH601628	11	0	0.00%	2	0	0.00%	0	0	/
SH601766	3	0	0.00%	4	1	25.00%	0	0	/
SH601818	5	0	0.00%	7	0	0.00%	0	0	/
SH600111	3	1	33.33%	2	0	0.00%	0	0	/
SH600837	13	0	0.00%	7	0	0.00%	0	0	/
SH601088	12	0	0.00%	7	1	14.29%	0	0	/
SH601186	9	0	0.00%	6	0	0.00%	0	0	/
SH601336	11	0	0.00%	4	1	25.00%	0	0	/
SH601398	16	0	0.00%	14	1	7.14%	0	0	/
Average	8	0	6.17%	5	0	9.69%	0	0	0.00%

Note: the average number of jumps and systematic co-jumps are rounded.

Table 2 indicates that under the traditional rule for identifying co-jumps, the number of co-jumps detected by traditional methods using a fixed sampling frequency is much lower than the number of actual co-jumps that we simulated. The average ratio of the detected co-jumps to the actual co-jumps hardly exceeds 40%, revealing that the traditional co-jump tests greatly underestimates the common risk and even underestimates the systematic risk. Under the updated rule for identifying co-jumps, the improved s-BNS, TOD and L-M tests can be used to correctly identify more co-jumps, and the former two gain an advantage over the latter in terms of accuracy and stability. The improved s-BNS correctly identifies 71.30% of the co-jumps on average, followed by the improved TOD, which identifies approximately 59.87% of the co-jumps. The results of the simulation experiment indicate that when jumps with persistence occur, the test procedure proposed in this study can be used to show its strength, while traditional tests urgently need to be improved (see Fig. 4).

4. Empirical results

The Shanghai Stock Exchange 50 Index (SSE 50 Index) is a selection of the 50 most representative stocks in the Shanghai stock market with large scale and good liquidity, and it is used to form sample stocks. We regard the SSE 50 Index in 2014 as a proxy for the

Table 4
Jumps in SSE 50 identified through BNS using both a fixed sampling starting point and traversed starting point.

Sampling frequency	Original BNS	BNS with traversed starting point
1 min	78	78
2 min	27	39
3 min	25	29
4 min	17	20
5 min	9	12
6 min	5	9
8 min	3	4
10 min	4	4
12 min	1	3
15 min	1	1
16 min	1	1
20 min	0	0
24 min	0	0
30 min	0	0

Table 5
Co-jump identified using the traditional test procedure.

Method	Stock	5 min			10 min			20 min		
		Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps
Original s-BNS	SH000016	188			103			55		
	SH600009	152	33	21.71%	144	22	15.28%	74	10	13.51%
	SH600016	302	31	10.26%	131	28	21.37%	82	19	23.17%
	SH600028	376	32	8.51%	187	23	12.30%	109	19	17.43%
	SH600030	293	77	26.28%	125	37	29.60%	63	20	31.75%
	SH600036	285	54	18.95%	118	24	20.34%	97	11	11.34%
	SH600048	299	50	16.72%	123	20	16.26%	79	20	25.32%
	SH600050	246	52	21.14%	136	21	15.44%	126	9	7.14%
	SH600104	265	41	15.47%	144	24	16.67%	79	11	13.92%
	SH600519	254	43	16.93%	129	26	20.16%	86	13	15.12%
	SH601166	301	44	14.62%	145	24	16.55%	94	13	13.83%
	SH600029	266	56	21.05%	197	29	14.72%	155	12	7.74%
	SH600887	268	39	14.55%	116	39	33.62%	76	20	26.32%
	SH601688	315	41	13.02%	177	20	11.30%	126	13	10.32%
	SH601901	312	64	20.51%	183	32	17.49%	108	15	13.89%
	SH601169	232	54	23.28%	169	33	19.53%	132	14	10.61%
	SH601328	264	40	15.15%	132	31	23.48%	104	17	16.35%
	SH601628	253	35	13.83%	116	23	19.83%	89	14	15.73%
	SH601766	299	43	14.38%	165	19	11.52%	125	12	9.60%
	SH601818	366	49	13.39%	184	26	14.13%	108	18	16.67%
	SH600111	232	76	32.76%	119	24	20.17%	89	16	17.98%
	SH600837	201	30	14.93%	128	18	14.06%	91	18	19.78%
	SH601088	223	33	14.80%	232	16	6.90%	167	17	10.18%
	SH601186	267	46	17.23%	176	36	20.45%	132	21	15.91%
	SH601336	316	52	16.46%	156	28	17.95%	101	22	21.78%
	SH601398	212	44	20.75%	197	21	10.66%	163	14	8.59%
	Average	269	46	17.47%	151	26	17.59%	104	16	15.76%
Original L-M	SH000016	46			18			13		
	SH600009	71	5	7.04%	44	2	4.55%	28	1	3.57%
	SH600016	266	16	6.02%	232	9	3.88%	71	5	7.04%
	SH600028	345	7	2.03%	301	2	0.66%	78	4	5.12%
	SH600030	126	24	19.05%	106	10	9.43%	55	11	20.00%
	SH600036	244	20	8.20%	149	11	7.38%	70	10	14.28%
	SH600048	314	17	5.41%	236	10	4.24%	87	8	9.19%
	SH600050	573	11	1.92%	559	6	1.07%	258	6	2.32%
	SH600104	237	27	11.39%	171	12	7.02%	108	8	7.40%
	SH600519	259	19	7.34%	214	11	5.14%	123	9	7.31%
	SH601166	302	33	10.93%	229	14	6.11%	131	12	9.16%
	SH600029	117	6	5.13%	87	3	3.45%	57	3	5.26%
	SH600887	189	17	8.99%	83	6	7.23%	48	4	8.33%
	SH601688	132	8	6.06%	115	3	2.61%	97	5	5.15%
	SH601901	269	22	8.18%	187	9	4.81%	146	8	5.48%
	SH601169	102	20	19.61%	79	5	6.33%	43	5	11.63%
	SH601328	289	11	3.81%	68	3	4.41%	36	2	5.56%
	SH601628	81	10	12.35%	84	4	4.76%	42	3	7.14%

(*continued on next page*)

Table 5 (*continued*)

Method	Stock	5 min			10 min			20 min		
		Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps
	SH601766	99	6	6.06%	85	2	2.35%	53	4	7.55%
	SH601818	123	13	10.57%	101	6	5.94%	88	4	4.55%
	SH600111	193	13	6.74%	60	4	6.67%	30	1	3.33%
	SH600837	258	16	6.20%	151	10	6.62%	107	2	1.87%
	SH601088	76	8	10.53%	54	6	11.11%	30	3	10.00%
	SH601186	128	12	9.38%	112	11	9.82%	88	3	3.41%
	SH601336	80	10	12.50%	59	4	6.78%	38	4	10.53%
	SH601398	70	6	8.57%	55	6	10.91%	49	3	6.12%
	Average	192	14	8.56%	140	7	5.73%	76	5	7.25%
Original TOD	SH000016	309			133			64		
	SH600009	381	80	21.00%	173	31	17.92%	89	11	12.35%
	SH600016	365	116	31.78%	191	48	25.13%	110	27	24.54%
	SH600028	404	53	13.12%	223	20	8.97%	133	14	10.52%
	SH600030	330	109	33.03%	165	55	30.30%	83	27	32.53%
	SH600036	340	112	32.94%	168	50	29.76%	125	32	25.60%
	SH600048	331	65	19.64%	185	37	20.00%	101	17	16.83%
	SH600050	287	36	12.54%	177	25	14.12%	226	15	6.63%
	SH600104	338	89	26.33%	189	46	24.34%	95	16	16.84%
	SH600519	328	62	18.90%	167	27	16.17%	108	15	13.88%
	SH601166	341	114	33.43%	198	66	33.33%	115	36	31.30%
	SH600029	321	28	8.72%	236	15	6.36%	230	15	6.52%
	SH600887	304	64	21.05%	154	27	17.53%	100	19	19.00%
	SH601688	374	54	14.44%	229	25	10.92%	200	15	7.50%
	SH601901	395	61	15.44%	210	26	12.38%	128	14	10.94%
	SH601169	314	64	20.38%	202	41	20.30%	237	20	8.44%
	SH601328	335	93	27.76%	176	44	25.00%	100	28	28.00%
	SH601628	318	53	16.67%	179	33	18.44%	311	20	6.43%
	SH601766	344	67	19.48%	206	40	19.42%	134	25	18.66%
	SH601818	402	53	13.18%	221	20	9.05%	133	14	10.53%
	SH600111	315	81	25.71%	153	35	22.88%	109	23	21.10%
	SH600837	268	37	13.81%	193	30	15.54%	314	14	4.46%
	SH601088	292	55	18.84%	282	26	9.22%	247	9	3.64%
	SH601186	326	47	14.42%	219	28	12.79%	173	18	10.40%
	SH601336	364	52	14.29%	179	23	12.85%	104	14	13.46%
	SH601398	308	41	13.31%	239	21	8.79%	347	16	4.61%
	Average	336	67	20.01%	194	34	17.66%	158	19	14.59%

portfolio and randomly selected 25 constituent stocks from those 50 stocks with complete high-frequency trading data as empirical objects. If there is an overlap between the jump intervals of the constituent stocks and the portfolio, then a systematic co-jump may occur. Table A1 in Appendix A lists the code that use to refer to these types of co-jumps in the following text. Cojumps that occurred during the 245 trading days of 2014 are mainly examined. All the data can be obtained from https://www.kaggle.com/datasets/shijiasong/price-data-of-sh50-in-2014.

In this section, we first use the results of the traditional BNS test to directly show the drawbacks of using a fixed sampling frequency and a fixed sampling starting point in the jump test. Then, we compare the empirical results of using the traditional s-BNS, L-M and TOD methods to those of using their improved versions (see Fig. 5).

4.1. Results of the BNS test

We use sampling frequencies of 5 min, 10 min and 20 min and apply the BNS test to these datasets. The significance level is set to 5%. The number of heterogeneous jumps and the proportion of systematic co-jumps are displayed in Table 3. It is worth noting that BNS can only support detection at the daily level.

Table 3 indicates that when the sampling interval is large, such as 20 min, it is usually difficult to identify a jump through BNS, which results in a large level of bias. Even under 5-minute or 10-minute sampling, the number of jumps detected in any stock or index does not exceed 20, and nearly half of the stocks show no evidence of co-jumps. Among all stocks, the average ratio of systematic co-jumps is less than 10% under these three sampling frequencies. The estimation bias can be explained through the persistence of jumps; if a jump occurs in both the index and constituent stock due to the same financial event on a specific day, but due to variations in persistence and starting point, they can hardly be detected through BNS since it is based on a fixed sampling pattern. This result drives the idea of considering jump persistence to adjust the estimation bias of the co-jump.

Under each sampling frequency in set $\{1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30\}$(minutes), we adopt two methods to perform the BNS test on SSE 50. One is to consider the first minute as the fixed sampling starting point, and the other is to consider each point in each sampling interval as the starting point. Table 4 shows the results of jump tests using these two different methods.

It can be seen from the table that at the same sampling frequency, the number of jumps detected by the BNS using a traversed starting point is relatively larger. The data displayed in Table 3 and Table 4 imply that both the sampling frequency and the sampling starting point affect the results of the jump test.

4.2. Results of the traditional jump test procedure

We apply traditional s-BNS, L-M and TOD to check the occurrence of co-jumps in 25 constituent stocks at the intraday level. Table 5 lists the results. The sampling frequencies are also chosen as 5 min, 10 min and 20 min.

Table 5 suggests that the higher the sampling frequency is, the more jumps that are detected and the higher the probability of systematic co-jumps in constituent stocks. Compared with the L-M method, s-BNS and TOD can be used to identify more jumps and systematic co-jumps. Under the 5-minute sampling frequency, approximately 20% of systematic co-jumps can be detected in individual stocks by TOD, but LM can only detect an average of 8.56% of the systematic co-jumps. Generally, if a jump is assumed to occur instantaneously, the proportion of systematic co-jumps in the constituent stocks is relatively low. The robustness of these results is also largely affected by the sampling frequency.

4.3. Results of the improved jump test procedure

Before performing the improved jump test procedure, the minimum sampling unit needs to be specified. The reason for emphasizing the so-called “minimum” is that when the sampling interval is large, jumps in opposite directions may occur within this time window, so that individual jumps in the opposite direction can be ignored and only persistent jumps with long time spans of persistence are identified, which somewhat underestimates the total number of jumps and overestimates the average time of jump persistence. This phenomenon can be found in the empirical data, and we illustrate it in Fig. 3 as an example. Fig. 3 displays the results of the s-BNS jump test for sh600104 from 9:30 a.m. to 10:00 a.m. on November 17, 2014. Using a sampling frequency of 2 min, a total of four consecutive jumps are identified, of which the first two are in the same direction. The third and the fourth are positive and negative jumps, respectively. According to the definition of jump persistence in this paper, the first two jumps can be regarded as a negative jump with a persistence time of 4 min and labeled jump B, while the last two jumps are labeled jumps C and D with a persistence time of 2 min each. At a sampling frequency of 5 min, a total of two jumps are identified, and these two jumps are consecutive and in the same direction, which means that we identify a negative jump A with a persistence time window of 10 min. By comparing the results at these two sampling frequencies, we find that when the minimum sampling interval increases, the time of persistence of individual jumps may be exaggerated to some extent, and the number of identified jumps is reduced. Accordingly, the number of co-jumps may be exaggerated because the larger time window of persistence creates a higher possibility of overlap with jumps across other assets.

Therefore, we choose a minimum sampling interval ranging from 1 to 5 min. We estimate the average time of jump persistence of the 25 constituent stocks in the SSE 50 using the improved jump tests mentioned in Section 2.4. We show the density plots of the average time of jump persistence based on 25 stocks under each of the three methods when the minimum sampling unit ranges from 1 min to 5 min. It is obvious that the lower that the sampling frequency is, the longer the average time of jump persistence will be, and the more homogeneous the value of the estimates. For example, when a 5-min sampling frequency is adopted, the average time of jump persistence is either 5 min or 10 min. As the sampling frequency increases, the average time of jump persistence gradually decreases

Table 6
Co-jump identified by the improved test procedure.

Stock	Improved s-BNS			Improved LM			Improved TOD		
	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps	Number of jumps	Number of Systematic Co-jumps	Ratio of Systematic Co-jumps
SH000016	122			263			406		
SH600009	209	40	19.14%	356	27	7.58%	376	81	21.54%
SH600016	314	66	21.02%	473	28	5.92%	414	145	35.02%
SH600028	399	83	20.80%	689	16	2.32%	439	51	11.62%
SH600030	176	144	81.82%	249	48	19.28%	353	164	46.46%
SH600036	304	54	17.76%	401	40	9.98%	336	130	38.69%
SH600048	387	93	24.03%	526	31	5.89%	239	76	31.80%
SH600050	412	113	27.43%	533	13	2.44%	381	32	8.40%
SH600104	266	145	54.51%	331	53	16.01%	448	114	25.45%
SH600519	312	70	22.44%	459	44	9.59%	364	100	27.47%
SH601166	402	83	20.65%	518	49	9.46%	406	138	33.99%
SH600029	354	96	27.12%	487	29	5.95%	234	25	10.68%
SH600887	225	87	38.67%	323	38	11.76%	345	89	25.80%
SH601688	424	67	15.80%	587	42	7.16%	323	35	10.84%
SH601901	289	122	42.21%	367	73	19.89%	480	70	14.58%
SH601169	364	65	17.86%	449	99	22.05%	208	47	22.60%
SH601328	267	98	36.70%	296	23	7.77%	439	122	27.79%
SH601628	199	78	39.20%	514	66	12.84%	205	32	15.61%
SH601766	287	68	23.69%	376	53	14.10%	360	68	18.89%
SH601818	354	72	20.34%	404	49	12.13%	418	52	12.44%
SH600111	236	86	36.44%	288	31	10.76%	380	107	28.16%
SH600837	113	66	58.41%	564	40	7.09%	138	23	16.67%
SH601088	197	51	25.89%	477	53	11.11%	208	26	12.50%
SH601186	199	59	29.65%	530	51	9.62%	230	33	14.35%
SH601336	208	58	27.88%	304	39	12.83%	295	49	16.61%
SH601398	572	64	11.19%	572	67	11.71%	129	23	17.83%
Average	292	81	30.43%	436	44	10.61%	329	73	21.83%

and shows a tendency to converge with the results from using a minimum sampling frequency of 2 min. The reason that the results do not converge to that of the 1-minute sampling frequency is probably that the high sampling frequency causes considerable noise that become identified as jumps, and these are mostly discrete and occur in different directions. Since only jumps in the same direction that occur continuously can be considered to be persistent jumps, the average time of jump persistence decreases significantly. Considering the convergence and the possible noise caused by the 1-min sampling frequency, we finally chose 2 min as the minimum sampling unit.

The involved parameters are consistent with the values used in the traditional L-M and TOD. Table 6 lists the results of the improved s-BNS, improved L-M and improved TOD methods.

Table 6 reveals that the improved test procedure based on the assumption that a jump has persistence can identify more jumps than the traditional test procedures. At the intraday level, the number and proportion of systematic co-jumps between constituent stocks and the index also increase greatly. Compared with the results obtained by applying the traditional methods with a sampling frequency of 5 min, the average proportion of co-jumps detected by the improved s-BNS is increased by approximately 13%. The improved L-M and the improved TOD methods only increase the average detection rate of systematic co-jumps by approximately 2% and 1.8%, respectively, over that of traditional tests. Although the improved method increases the sampling frequency to a high level, the ratio of co-jumps does not increase to a corresponding degree since it only regards consecutive jumps in the same direction as a single jump and regards jumps whose time windows of persistence overlap as co-jumps. This means that the improved method can increase the proportion of detected co-jumps to a certain degree while simultaneously reducing the influence of market microstructure noise on co-jump detection when using a high sampling frequency. However, the proportion of intraday systematic co-jumps detected by the improved L-M and TOD is still lower than that of intraday systematic co-jumps detected by the improved s-BNS. The proportion of systematic co-jumps obtained by the improved s-BNS reaches approximately 30% on average, which indicates that co-jumps and even systematic co-jumps are not sparse. Since the improved s-BNS method more accurately identifies co-jumps in simulation experiments, the empirical test results of the improved s-BNS may be more convincing, which indicates that systematic risk occurs more frequently than previously thought.

To make the test results of the improved procedures and those of the traditional methods more comparable, we attempted to use the 5-min minimum sampling unit as well, and the details are displayed in Table A2 of the Appendix. The results indicate that the number of jumps identified by the 5-minute minimum sampling interval decreases relative to that of the 2-minute minimum sampling interval, yet the number and proportion of systematic co-jumps increases, which we consider to be less convincing based on the above theoretical analysis. Hence, these results are only shown in the Appendix as a reference.

5. Conclusion

We begin with the phenomenon that jumps have a certain persistence and reveal the limitations of traditional jump tests, such as

BNS, s-BNS, L-M, and TOD, in detecting these jumps. Since the common rule used to define co-jumps is also based on the assumption of jump synchronization, we believe that the existing scheme of detecting co-jumps underestimates systematic risks to a large extent, which in turn poses a challenge to risk management. To improve the procedure of identifying co-jumps, on the one hand, we improve the tests to allow for the detection of jumps with persistence at both the daily and intraday levels. On the other hand, we extend the coexceedance rule for defining co-jumps (Gilder et al., 2014) to make it more consistent with reality.

We conduct a simulation experiment to compare the accuracy of the traditional jump test procedure and the improved jump test procedure in determining co-jumps. The results show that the improved jump test procedure has much higher accuracy than the traditional jump test procedure, with the improved s-BNS method showing the relative best performance. We then regard the Shanghai Stock Exchange 50 Index as a proxy of the portfolio and check the frequency of systematic co-jumps in its 25 constituent stocks. The empirical results show that the traditional s-BNS, TOD, and L-M methods cannot identify jumps well when using certain fixed sampling frequencies and starting points. At the intraday level, with a sampling frequency of 5 min, the average proportions of co-jumps detected by the traditional test procedure are approximately 20% at most. While the improved s-BNS detects approximately 30% of systematic co-jumps on average in these constituent stocks, the improved TOD and the improved L-M also detect more systematic co-jumps than the traditional TOD and L-M, which indicates that co-jumps and even systematic co-jumps are not sparse at all, and current jump test procedures generally underestimate risk.

This study demonstrates the persistence of jumps and the effectiveness of jump tests that consider different sampling frequencies and starting points. Such a scheme in the detection of co-jumps could improve the estimation accuracy of systematic risk and reveal higher nondiversifiable risks between market indices and individual stocks, thereby providing new directions for risk management. In the future, on the basis of jump persistence, we also expect to propose a new generation mechanism for co-jumps, which includes information regarding their starting point, persistence and amplitude, to further complete related studies.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

I have shared the link to my data at the Attach Files step

Appendix A

Table A1
Empirical objects and their codes.

Stock	Code
SSE 50	SH000016
Shanghai International Airport	SH600009
Minsheng Bank	SH600016
Sinopec	SH600028
CITIC Securities	SH600030
China Merchants Bank	SH600036
Poly Real Estate	SH600048
China Unicom	SH600050
SAIC	SH600104
Guizhou Moutai	SH600519
Industrial Bank	SH601166
China Southern Airlines	SH600029
Yili Industrial Group	SH600887
Huatai Securities	SH601688
Founder Securities	SH601901
Bank of Communications	SH601328
China Life Insurance Company	SH601628
CRRC Corporation Limited	SH601766
China Everbright Bank	SH601818
China Northern Rare Earth Group	SH600111
Bank of Beijing	SH601169
Haitong Securities	SH600837
China Shenhua Energy	SH601088
China Railway Construction	SH601186
New China Life Insurance	SH601336
Industrial and Commercial Bank of China	SH601398

Table A2
Co-jump identified by improved test procedure (at 5-min sampling frequency).

Stock	Improved s-BNS			Improved LM			Improved TOD		
	Number of jumps	Number of Systemic Co-jumps	Ratio of Systemic Co-jumps	Number of jumps	Number of Systemic Co-jumps	Ratio of Systemic Co-jumps	Number of jumps	Number of Systemic Co-jumps	Ratio of Systemic Co-jumps
SH000016	101			40			267		
SH600009	187	44	23.53%	68	36	52.94%	364	85	23.35%
SH600016	297	60	20.20%	245	41	16.73%	323	132	40.87%
SH600028	365	79	21.64%	289	29	10.03%	386	49	12.69%
SH600030	144	146	101.39%	101	48	47.52%	304	146	48.03%
SH600036	276	60	21.74%	204	53	25.98%	298	114	38.26%
SH600048	221	101	45.70%	301	39	12.96%	224	79	35.27%
SH600050	265	109	41.13%	397	56	14.11%	269	39	14.50%
SH600104	254	149	58.66%	209	47	22.49%	303	97	32.01%
SH600519	232	79	34.05%	215	43	20.00%	287	86	29.97%
SH601166	286	87	30.42%	289	67	23.18%	306	124	40.52%
SH600029	245	93	37.96%	107	46	42.99%	226	47	20.80%
SH600887	197	89	45.18%	183	49	26.78%	275	76	27.64%
SH601688	308	74	24.03%	119	56	47.06%	312	44	14.10%
SH601901	264	135	51.14%	243	85	34.98%	359	77	21.45%
SH601169	245	69	28.16%	98	65	66.33%	195	49	25.13%
SH601328	241	101	41.91%	246	44	17.89%	297	109	36.70%
SH601628	203	83	40.89%	76	34	44.74%	187	59	31.55%
SH601766	232	74	31.90%	93	24	25.81%	306	69	22.55%
SH601818	310	76	24.52%	101	43	42.57%	375	54	14.40%
SH600111	208	89	42.79%	181	29	16.02%	279	102	36.56%
SH600837	97	78	80.41%	232	43	18.53%	119	36	30.25%
SH601088	165	48	29.09%	73	26	35.62%	196	38	19.39%
SH601186	156	63	40.38%	109	23	21.10%	212	46	21.70%
SH601336	167	65	38.92%	76	37	48.68%	276	58	21.01%
SH601398	43	59	137.21%	68	45	66.18%	109	40	36.70%
Average	220	84	38.18%	168	44	26.19%	271	74	27.31%

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